Hawk04-D From Finite Enter Interfaces to Conditional Evaporation Certificates: Controlled Limits, Mass Laws, and Quantum Recovery

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Abstract

We develop a proof-based extrapolation framework for the finite Enter interfaces of Hawk04-C. The operational core consists of a complete isometry, declared accessible and inaccessible outputs, quantum-return decoders, destructive readout, and explicit classical discard. We prove exact calibration formulas for a partial-SWAP schedule and a flagged sequential-transfer model, including their different information targets. We establish fixed-code approximation and growing-code certification statements, reconstruct a finite-band reservoir regulator, and give an independent thermal-pair truncation certificate. The regulator is not transferred to a collapsing-shell Bogoliubov model without a separate comparison theorem.

A supplied luminosity law is treated as an additional physical hypothesis. Under fixed massless-species and greybody scaling assumptions, we derive the inverse-square power law and the resulting cubic cutoff-lifetime formula, together with perturbation and discretization bounds. These conclusions stop at a positive validity cutoff and do not determine Planck-scale behavior. Entropy turnover, approximately thermal observations, complementary decoupling, and full-code recovery are separated by explicit counterexamples and dimension constraints. An access-preserving endpoint package yields a fixed-decoder certificate; terminal residual readout-and-discard leaves the existing radiation channel unchanged.

The resulting predictions are conditional on independently supplied dynamics, access, approximation, luminosity, and endpoint hypotheses. No gravitational evaporation channel, universal Page-time recovery transition, final-burst spectrum, or absence of physical remnants is derived. The contribution is a coherent certification architecture and a set of calibrated tests, not a solution of black-hole evaporation.

Bibliographic record

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Central principle. A finite channel theorem can certify a physical prediction only after the physical model supplies a comparison at that channel interface. Thermal matching, a mass equation, and residual decoupling are separate hypotheses. None is supplied by the word ``Enter."

Background, Enter philosophy, and the corrected extrapolation

The physical background

Hawking's particle-creation calculation concerns quantum fields on a classical collapse background, while his information-loss proposal asks about a complete final description of the initial quantum state

[Hawking1975,Hawking1976]. For an uncharged, nonrotating Schwarzschild black hole, the external semiclassical temperature is

$$T_{\mathrm{H}} = \frac{\hbar c^3}{8\pi G M k_{\mathrm{B}}}.$$

A bosonic mode has a thermal denominator modified by an absorption factor:

$$\langle n_{\omega \ell m} \rangle = \frac{\Gamma_{\omega \ell m}}{\exp\left(\hbar\omega/(k_{\mathrm{B}} T_{\mathrm{H}})\right) - 1}.$$

These statements do not specify a complete logical-input-to-radiation channel. In particular, one occupation number is not a test of every logical coherence.

Page's random-subsystem analysis and unitary-radiation comparison concern additional assumptions about states and information in radiation [PageAverage,PageInformation]. Random-subsystem recovery also depends on the input message, initial correlations, and subsystem sizes, rather than on a universal count of ``half the emitted quanta'' [HaydenPreskill]. Island and replica-wormhole calculations address entropy in their specified gravitational settings [AlmheiriEtAl]. They are not replaced here by a finite circuit, and they are not used as a substitute for a missing complete-channel comparison.

The low-energy scalar-reservoir evaporation model of Gaikwad, Kaushal, Mandal, and Wadia has its own dynamics and physical assumptions [GaikwadEtAl]. Its geometry is not assigned to the finite-band source model below merely because both contain a scalar bath.

The Philosophy of Enter

Cheung's *Philosophy of Enter* treats measurement as a declared interface: destructive computational-basis readout consumes the measured quantum name and returns classical data. It does not return the unknown quantum input with its reference correlations. A decoder has a different target and must specify its permitted access. The essay also separates complete energy, physical erasure cost, and residual leakage, and distinguishes an operational analogy from a physical identification [CheungPhilosophy].

We implement that discipline by declaring the input code, its retained references, the accessible output, every excluded output, and the target channel before stating a recovery or observation result. Register consumption is a statement about the returned interface, not a proof that a physical environment has disappeared.

Cheung's journal article develops a linear quantum lambda calculus and an operational characterization of destructive measurement within an explicit strategy class [CheungEnter]. Here the finite Born instrument is defined directly. We neither reproduce that uniqueness argument nor enlarge its hypotheses by citation.

The analogy and its added obligations

In the finite analogy:

• a prepared quantum code plays the role of an information-bearing source;

• the split $R \mid B$ specifies an access restriction;

• a transfer isometry exposes a new outgoing quantum factor;

• Enter converts a chosen output into a classical record;

• a terminal instruction consumes a named interface. These are operational descriptions. A metric, horizon, stress tensor, radiation flux, and physical endpoint require additional constructions.

Hawk04-C supplies the six-row operational separation, while Hawk04-A and Hawk04-B supply complete-complement recovery, finite-regulator certificates, and an interacting-family negative control [CheungHawk04C,CheungHawk04A,CheungHawk04B]. The supplied excerpts of Hawk02-A, Hawk02-B, Hawk01-B, and En07-A/B/C motivate the staged organization used here [CheungHawk02A,CheungHawk02B,CheungHawk01B, CheungEn07A,CheungEn07B,CheungEn07C]. All elementary statements needed below have local proofs.

Corrections to the proposed plan. The following repairs are substantive.

• The general complementary-Choi recovery allowance is $\sqrt{(2de)} + b$, not $2de + b$. A linear bound requires a

different, independently justified certificate.

• A sequence of changing input dimensions does not have a well-defined diamond comparison until input embeddings and recovery targets are specified.

• The finite-band reservoir regulator is model specific. It is not a regulator theorem for a collapsing-shell Bogoliubov transformation.

• An arbitrary shrinking core does not imply decoupling of the complete residual. A whole *n*-qubit code and a small injected diary are different recovery tasks.

• Temperature matching alone does not determine a luminosity, emission clock, or lifetime.

• Entropy turnover does not establish a referenced decoder or fix its transition time.

• A fixed information-free residual can have nonzero energy and many physical degrees of freedom. It is not synonymous with no remnant.

• A null detector result is not, without a statistical and population model, proof that remnants do not exist.

The exact channel principles have established antecedents in Stinespring dilation, quantum error correction, no-hiding, and information–disturbance theory [Stinespring,KnillLaflamme,BraunsteinPati,KSW]. No priority claim is made for those principles.

Complete interfaces and the finite operational core

Definitions

Let $C \simeq \mathbb{C}\hat{}d$, $d \geq 2$, be the entire logical input whose identity channel is the recovery target. Output Hilbert spaces may be separable; finite output spaces are stated explicitly whenever finite-dimensional optimization is used. Channels act on trace-class operators and are normal CPTP maps. An external reference Q is untouched.

Definition. A complete interface is an isometry

$$V : C \to R \otimes B, \qquad V^{\dagger}V = I_C.$$

Its complete, accessible, and complementary channels are

$$\mathcal{V}(X) = VXV^{\dagger}, \qquad \mathcal{M}(X) = \mathrm{Tr}_B\, \mathcal{V}(X), \qquad \mathcal{E}(X) = \mathrm{Tr}_R\, \mathcal{V}(X).$$

The complete inaccessible output B includes all unreturned preparation purifiers, source registers, uncollected modes, apparatus outputs, work registers, and discarded-record stores in the declared model. The independent reference Q is not part of B.

For a state σ, write

$$\mathcal{T}_{\sigma}(X) = \mathrm{Tr}\,(X)\sigma.$$

Use half-normalized distances

$$d_{\mathrm{tr}}(\rho,\eta) = \frac{1}{2}\|\rho - \eta\|_1, \qquad d_{\diamond}(\Phi,\Psi) = \frac{1}{2}\|\Phi - \Psi\|_{\diamond},$$

and define

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}) = \inf_{\mathcal{D} : R \to C \ \mathrm{CPTP}} d_{\diamond}(\mathcal{D}\mathcal{N}, \mathrm{id}_C),$$
$$\delta_{\mathrm{full}}(\mathcal{E}) = \inf_{\sigma \in \mathcal{D}(B)} d_{\diamond}(\mathcal{E}, \mathcal{T}_{\sigma}).$$

Decoders are deterministic and return a quantum code. A normalized successful branch alone is not a decoder.

A reference of dimension d suffices for differences of channels with input C, by Schmidt decomposition and convexity. Consequently all diamond statements below retain arbitrary references.

Theorem. For a complete interface, choose any $\tau_C \in \mathcal{D}(C)$. The map

$$\mathcal{L}_V(Y) = V^{\dagger}YV + \mathrm{Tr}\,((I - VV^{\dagger})Y)\tau_C$$

is CPTP and satisfies $\mathcal{L}_V\mathcal{V} = id_C$. Isometric embedding preserves the trace norm of every trace-class operator, including after tensoring with I_Q.

Moreover, the following are equivalent:

• a CPTP decoder satisfies $\mathscr{D}\mathscr{N} = id_C$;

• $\mathscr{E} = \mathscr{T}_\{\sigma_B\}$ for one residual state;

• $\mathscr{E}(|i\rangle\langle j|) = \delta_ij\sigma_B$ on all matrix units;

•

$$J_{\mathrm{n}}(\mathscr{E}) = I_Q/d \otimes \sigma_B, \qquad J_{\mathrm{n}}(\Phi) : = (\mathrm{id}_Q \otimes \Phi)(|\Phi_d\rangle\langle\Phi_d|),$$

where $|\Phi_d\rangle = d\^\{-1/2\}\sum_i |i\rangle_Q|i\rangle_C$. Under these conditions, there are a space K, a state σ_K, and an isometry $A: C \otimes K \to R$ such that

$$\mathscr{N}(X) = A(X \otimes \sigma_K)A^\dagger.$$

An exact decoder on all of R is

$$\mathscr{D}(Y) = \mathrm{Tr}_K\,(A^\dagger Y A) + \mathrm{Tr}\,((I - AA^\dagger)Y)\tau_C.$$

If σ_B has finite rank r, then $dim\ R \geq dr$.

Proof. Compression and positive-effect measurement followed by preparation are CP. Their traces in add to $Tr\ Y$, and direct substitution gives the left inverse. An isometry preserves all nonzero singular values, proving trace-norm preservation.

Matrix units span $\mathscr{L}(C)$, and their Bell expansion proves the equivalence of (ii)–(iv). Assume (ii), and diagonalize the nonzero residual support:

$$\sigma_B = \sum_\alpha p_\alpha |b_\alpha\rangle\langle b_\alpha|, \qquad p_\alpha > 0.$$

The index set is finite or countable. Constancy implies that all code vectors are supported in $R \otimes supp\ \sigma_B$, so write

$$V|i\rangle = \sum_\alpha \sqrt{p_\alpha}\, |u_{i\alpha}\rangle_R |b_\alpha\rangle_B.$$

The complementary matrix-unit identities give

$$\langle u_{j\beta}, u_{i\alpha}\rangle = \delta_{ij}\delta_{\alpha\beta}.$$

Define $A(|i\rangle|\alpha\rangle) = |u_\{i\alpha\}\rangle$ and $\sigma_K = \sum_\{\alpha\}\ p_\{\alpha\}|\alpha\rangle\langle\alpha|$. These vectors are orthonormal, proving the normal form and the dimension bound. The two terms in are CP and have traces summing to $Tr\ Y$; substitution returns X.

Conversely, dilate an exact decoder by a Stinespring isometry $L: R \to C \otimes G$ [Stinespring]. The composite $Z = (L \otimes I_B)V$ has pure decoded output $|i\rangle$ on input $|i\rangle$, hence

$$Z|i\rangle = |i\rangle|\xi_i\rangle_{GB}.$$

Recovery of each off-diagonal matrix unit requires $\langle\xi_j, \xi_i\rangle = 1$. All ξ_i are one vector ξ. Tracing the decoded output and G gives

$$\mathscr{E}(|i\rangle\langle j|) = \delta_{ij}\mathrm{Tr}_G\,|\xi\rangle\langle\xi|.$$

Every equality holds on all operators and therefore on all reference extensions.

This is the full-input constant-complement structure of exact correction [KnillLaflamme,BraunsteinPati]; compare the locally proved formulations in Hawk04-A/B/C [CheungHawk04A,CheungHawk04B, CheungHawk04C].

Corollary. The residual alone recovers the whole code exactly if and only if the accessible channel is a replacement channel. A merely nonconstant complement obstructs exact recovery from R; it does not establish recovery from B. For $d \geq 2$, both factors cannot independently recover the whole code.

Proof. Interchange the output factors in Theorem . A replacement channel cannot recover two distinct inputs.

Enter, discard, and finite soundness

Let S be a measured finite register and T the surviving quantum register. In the computational basis, put $M_x = \langle x|_S \otimes I_T$. Define the destructive instrument

$$\mathscr{M}_S(X) = \sum_x |x\rangle\langle x|_K \otimes M_x X M_x^\dagger.$$

The output names are *K*, *T*, not *S*; *K* carries the diagonal classical algebra. Define

$$\mathrm{dropBit}_K\Big(\sum_x |x\rangle\langle x|_K \otimes X_x\Big) = \sum_x X_x.$$

Proposition. The instrument is CPTP and

$$\mathrm{dropBit}_K \mathscr{M}_S = \mathrm{Tr}_S.$$

For any CPTP operation on *B*, with all its outputs ignored, the untouched *QR* marginal is unchanged. Thus residual Enter followed by Drop leaves $\mathscr{N}$ and *ε_rec(𝒩)* exactly unchanged.

A complete readout dilation is

$$W_{\mathrm{read}}|\psi\rangle = \sum_x |x\rangle_K |x\rangle_E \otimes M_x|\psi\rangle.$$

It is an isometry. Tracing *E* gives . Discarding *K* may be represented by moving its carrier to another inaccessible store. Those outputs remain in a complete dilation.

Proof. The measurement bras satisfy *∑_x M_x^{†}M_x = I*. Summing the classical blocks gives the basis formula for partial trace. For a local channel *Λ_B*, its adjoint is unital, so every untouched observable *A* obeys

$$(\mathrm{id} \otimes \Lambda_B^*)(A \otimes I) = A \otimes I.$$

All untouched expectations agree, including references. Orthogonality of the two outcome labels and measurement completeness give *W_read^{†}W_read = I*. Tracing *E* removes the cross-outcome terms.

Conditional states can change when an outcome is supplied. That is a different access task. An observer restricted to *R* also cannot execute Enter on *B* without an additional permission [CheungHawk04B,CheungHawk04C,CheungEn07A].

Proposition. A finite acyclic instruction tree built from specified pure allocation, specified unitary constants, destructive Enter, classical control, classical discard, and a fixed returned hybrid type denotes a CPTP channel. Every quantum name is returned or explicitly consumed. There are refined path maps *K_h* with

$$\sum_h K_h^\dagger K_h = I,$$

and each reported output block is the sum of its unnormalized path maps *X ↦ K_hXK_h^{†}*. The denotation retains arbitrary references, and every branch terminates.

Proof. Allocation is isometric, unitaries are trace preserving, and readout bras are complete. Classical control selects a complete continuation on each block; discard sums blocks. Backward induction on the finite tree proves path completeness. Grouping histories proves the block formula. Tensoring path maps with *I_Q* proves the reference statement. A finite tree has finite maximum branch length.

The theorem neither compiles the constants over a fixed gate library nor proves a physical endpoint. It uses the finite ownership discipline of Hawk04-B/C and the separate sequential-accounting comparison in En02-C [CheungHawk04B,CheungHawk04C,CheungEn02C].

Two sharp benchmarks

Set *a_d = 1 − d^{−2}*.

Lemma. A replacement accessible channel has optimal recovery error *a_d*. A measure-and-prepare channel, followed by any decoder to *C*, has error at least *1 − d^{−1}*. Computational readout and basis-state preparation attain the latter bound.

Proof. A decoded replacement has Bell output *I_Q/d ⊗ τ_C* and Bell overlap *d^{−2}*. The Bell-projector test gives error at least *a_d*. Preparing *I_C/d* attains it: the completely depolarizing channel is the uniform average of the *d^2* Weyl unitary channels, and separating the identity term gives the matching upper bound.

A measure-and-prepare channel has separable Bell output. A normalized product vector has squared overlap with *Φ_d* at most *1/d*, by Cauchy–Schwarz; convexity extends this to separable states. The Bell test gives *1 − 1/d*.

Computational dephasing is the uniform average of the *d* phase unitaries *Z_d^k*. Its Bell outputs are orthogonal, giving diamond distance *1 − 1/d* from the identity.

Measure-and-prepare channels are entanglement breaking [HorodeckiShorRuskai]. The benchmark does not prohibit classical assistance when a surviving quantum output is also returned [CheungHawk04B,CheungHawk04C].

Stage 1: two different finite calibration schedules

The whole-code SWAP schedule

Let $C = C_A \otimes C_D$, with dimensions *a*, *m*, so $d = am$. The first *k* transferred qubits of an *n*-qubit SWAP schedule correspond to $a = 2^k$, $m = 2^{n-k}$. Before measurement, the accessible channel is

$$\mathcal{N}_k = \mathrm{Tr}_{C_D},$$

up to fixed output dictionaries and blank factors. The complement returns *C_D*.

Theorem. For this whole-code target,

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}_k) = \delta_{\mathrm{full}}(\mathcal{E}_k) = 1 - m^{-2}.$$

If only the computational record on *C_A* is returned, the channel is

$$\mathcal{F}_k(X) = \sum_{x=1}^{a} \mathrm{Tr}\left((|x\rangle\langle x| \otimes I_m)X\right)|x\rangle\langle x|,$$

and its optimal recovery error is

$$\varepsilon_{\mathrm{rec}}(\mathcal{F}_k) = 1 - \frac{1}{am^2}.$$

Thus, for *n* qubits,

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}_k) = 1 - 2^{-2(n-k)}, \qquad \varepsilon_{\mathrm{rec}}(\mathcal{F}_k) = 1 - 2^{-(2n-k)}.$$

These are analytic identities, not reported numerical executions.

Proof. The decoder returning *C_A* and preparing *I_C_D/m* has decoded channel *id_C_A ⊗ T_{I/m}*, with error *1 − m^{−2}*. For the lower bound, use independent Bell states for *C_A*, *C_D*. After *Tr_C_D*, the reference *Q_D* is *I/m* and independent of everything available to the decoder. The decoded full Bell overlap is at most *m^{−2}*: tracing *Q_D* against its Bell projector gives *I_C_D/m^2*, and the remaining Bell-projector expectation is at most one. This proves the recovery lower bound.

The complementary channel is *Tr_C_A* with output *C_D*. Choosing replacement *I/m* gives leakage at most *1 − m^{−2}*. Fixing *C_A* and testing a Bell state on *C_D* gives the reverse bound by Lemma .

For the record channel, write $F_x = |x\rangle\langle x| \otimes I_m$ and let σ_x be the decoder output on record *x*. Its Bell overlap is

$$\frac{1}{d^2}\sum_{x=1}^{a} \mathrm{Tr}\left(F_x \sigma_x\right) \le \frac{a}{d^2} = \frac{1}{am^2}.$$

For attainability, prepare $|x\rangle\langle x| \otimes I_m/m$. The decoded channel is $\Delta_a \otimes \mathcal{T}_{I/m}$, the uniform mixture of am^2 orthogonal-Bell-output unitary channels, one of which is the identity. Its distance from the identity is $1 - 1/(am^2)$.

In particular, emitting half the qubits of a completely unknown *n*-qubit input is not enough to recover that whole input. Immediate readout makes the task stricter still. This calibrates the schedule of Hawk04-C without confusing a whole star code with one small diary [CheungHawk04C].

A single logical cell with an occupied-vacuum flag

Let a memory cell and each outgoing cell contain $\mathbb{C}|vac\rangle \oplus C$. At step *j*, a specified unitary rotates

$$|i\rangle_M|\mathrm{vac}\rangle_{S_j} \mathrm{longmapstocos}\,\theta_j|i\rangle_M|\mathrm{vac}\rangle_{S_j} + \sin\theta_j|\mathrm{vac}\rangle_M|i\rangle_{S_j}$$

for all logical labels *i*, using the complementary rotation on the opposite occupied-vacuum states and the identity elsewhere. Choose $0 \le \theta_j \le \pi/2$. Define

$$q_j = \prod_{\ell=1}^{j} \cos^2\theta_\ell, \qquad q_0 = 1.$$

The amplitude of first transfer at step *s* is *sin θ_s∏_{ℓ < s} cos θ_{ℓ}*. Orthogonal transfer locations imply that, when *q_j < 1*, these amplitudes define an isometry *A_j: C → R_j* after normalization.

Theorem. After *j* pulses, the complete map has the form

$$V_{q_j}|\psi\rangle = \sqrt{1-q_j}\,A_j|\psi\rangle|\mathrm{vac}\rangle_B + \sqrt{q_j}\,|\mathrm{vac}\rangle_R L|\psi\rangle,$$

where the occupied ranges of *A_j*, *L* are orthogonal to their vacuum flags. Its optimal recovery and leakage are

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}_j) = \delta_{\mathrm{full}}(\mathscr{E}_j) = q_j a_d.$$

An optimal decoder compresses the occupied radiation code and prepares *I_C/d* on its orthogonal complement.

If

$$H_0 = \omega\Big(\Pi_{\mathrm{occ},M} + \sum_s \Pi_{\mathrm{occ},S_s}\Big),$$

each pulse conserves *H_0*. For one initially occupied cell,

$$E_{\mathrm{rad}}(j) = \omega(1-q_j).$$

For *K* independent cells with the same *q_j*, this expectation is *Kω(1 − q_j)*, even for correlated logical labels.

Proof. Induction gives the pulse amplitudes above, and

$$\sum_{s=1}^{j} \sin^2\theta_s \prod_{\ell<s} \cos^2\theta_\ell = 1 - q_j.$$

This proves and its isometry property.

The accessible channel is

$$\mathcal{N}_j(X) = (1-q_j)A_jXA_j^\dagger + q_j\mathrm{Tr}\,(X)|\mathrm{vac}\rangle\langle\mathrm{vac}|.$$

The stated decoder produces *(1 − q_j)id_C + q_j𝒯_{I/d}*, with error *q_ja_d*. Any decoder's Bell overlap is at most *(1 − q_j) + q_j/d^2*, proving optimality.

The complement is

$$\mathscr{E}_j(X) = q_jLXL^\dagger + (1-q_j)\mathrm{Tr}\,(X)|\mathrm{vac}\rangle\langle\mathrm{vac}|.$$

Replacement *q_jLI_CL^{†}/d + (1 − q_j)|vac⟩⟨vac|* gives leakage at most *q_ja_d*. For an arbitrary replacement state, apply a CPTP pinching and retraction onto the occupied code block and vacuum flag. Let *u* be its occupied weight. On a Bell input, the occupied Bell projector has expectation difference *q_j − u/d^2*. The effect equal to that projector on the occupied block and identity on vacuum has difference *ua_d*. If *u ≤ q_j*, the first is at least *q_ja_d*; if *u ≥ q_j*, the second is at least *q_ja_d*. This proves the leakage lower bound.

Each pulse mixes equal-energy one-excitation states and is the identity on all other sectors. It commutes with *H_0*. The transfer probability is *1 − q_j*, proving the energy formula. Linearity and addition of the independent number observables prove the *K*-cell statement.

These formulas reconstruct the single-cell benchmark reported in the supplied Hawk02-B excerpt and the flagged comparison in Hawk04-A [CheungHawk02B,CheungHawk04A]. They do not describe the whole-code partial-trace channel of Theorem .

Corollary. Every nonincreasing sequence *1 = q_0 ≥ q_1 ≥ ⋯ ≥ q_N ≥ 0* is realized by the pulses above. Thus a sharp leakage decrease near a chosen step is a realizable design choice, not a derived Page-time law.

Proof. When *q_{j − 1} > 0*, take *cos^2 θ_j = q_j/q_{j − 1}*. After *q_j = 0*, later pulses can be trivial.

Bare-energy conservation here is not a complete switching-work or cyclic controller-reset certificate. Full computational Enter on the logical labels also destroys their quantum coherence; measuring only an occupation flag is a different instrument.

Stage 2: controlled comparison, fixed codes, and growing codes

Stability and dimension-explicit conversion

Lemma. On common output spaces,

$$|\varepsilon_{\mathrm{rec}}(\mathcal{N}_1) - \varepsilon_{\mathrm{rec}}(\mathcal{N}_2)| \mathrm{led}_{\diamond}(\mathcal{N}_1, \mathcal{N}_2), \qquad |\delta_{\mathrm{full}}(\mathscr{E}_1) - \delta_{\mathrm{full}}(\mathscr{E}_2)| \mathrm{led}_{\diamond}(\mathscr{E}_1, \mathscr{E}_2).$$

Separate isometric embeddings of R and B preserve both diagnostics.

For common-output isometries,

$$d_{\diamond}(\mathcal{V}_1, \mathcal{V}_2) \le \|V_1 - V_2\|_{\infty}.$$

For channels with input dimension d,

$$d_{\diamond}(\Phi, \Psi) \le \min\left\{1, \frac{d}{2}\|J_{\mathrm{n}}(\Phi) - J_{\mathrm{n}}(\Psi)\|_1\right\}.$$

Proof. For each decoder, the triangle inequality and CPTP contraction change its error by at most the channel comparison distance. Take infima in both directions. Distance to the replacement set is likewise one-Lipschitz.

An isometry $J: H \to H^{'}$ has the CPTP retraction

$$Y \mathrm{longmapsto} J^{\dagger} Y J + \mathrm{Tr}\left((I - JJ^{\dagger})Y\right)\tau_H.$$

Embedding candidate decoders or replacement states and then retracting proves invariance.

For referenced input ρ, expand the complete-output difference as

$$(V_1 - V_2)\rho V_1^{\dagger} + V_2 \rho (V_1^{\dagger} - V_2^{\dagger}),$$

with reference identities understood. Its trace norm is at most $2\|V_1 - V_2\|_{\infty}$. Partial trace is contractive.

Every pure input with a d-dimensional reference is $(L \otimes I)|\Phi_d\rangle$, with $\mathrm{Tr}(L^{\dagger}L) = d$ and $\|L\|_{\infty}^2 \le d$. Its output difference is the Choi difference multiplied on both sides by L, $L^{\dagger}$. Convexity completes the bound.

These stability statements agree with the supplied Hawk02-A interface and Hawk04-A/B proofs [CheungHawk02A,CheungHawk04A, CheungHawk04B].

The external approximate input. For finite-dimensional complementary channels, the KSW information–disturbance theorem gives

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}) \le \min\{1, \sqrt{2\delta_{\mathrm{full}}(\mathscr{E})}\}.$$

Indeed the full-norm inequality

$$\frac{1}{4}\Big(\inf_{\mathscr{D}} \|\mathscr{D}\mathcal{N} - \mathrm{id}_C\|_{\diamond}\Big)^2 \le \inf_{\sigma} \|\mathscr{E} - \mathscr{T}_{\sigma}\|_{\diamond}$$

becomes under the half normalization [KSW]. We use this approximate theorem only on finite channels. No infinite-output attainment assumption is needed for the proxy lift.

Theorem. Let $\mathcal{V}: C \to R \otimes B$ be a finite isometry, embedded factorwise into the output space of V. Suppose

$$d_{\diamond}(\mathcal{V}, \widehat{\mathcal{V}}) \le b.$$

For a specified proxy residual state $\hat{\sigma}$, set

$$e = \frac{1}{2}\left\|J_{\mathrm{n}}(\widehat{\mathscr{E}}) - I_Q/d \otimes \hat{\sigma}\right\|_1.$$

Then

$$\delta_{\mathrm{full}}(\mathscr{E}) \le \min\{1, de + b\}.$$

A finite decoder, extended CPTP to all of R, has error

$$d_{\diamond}(\mathscr{D}\mathcal{N}, \mathrm{id}_C) \le \min\{1, \sqrt{2de} + b\}.$$

Consequently $\varepsilon_{rec}(\mathcal{N})$ is also at most a_d.

For a specified proxy decoder with

$$r_{\mathrm{dec}} = \frac{1}{2}\left\| J_{\mathrm{n}}(\widehat{\mathscr{D}\mathscr{N}}) - |\Phi_d\rangle\langle\Phi_d| \right\|_1,$$

its extension satisfies

$$d_\diamond(\mathscr{D}\mathscr{N}, \mathrm{id}_C) \le \min\{1, d r_{\mathrm{dec}} + b\}.$$

Proof. The complete comparison gives allowance b for both reduced channels. Equation bounds proxy leakage by *de*. The triangle inequality proves . Finite decoder Choi matrices form a compact set, so an optimal proxy decoder exists. Apply to the proxy.

Let *P_R* project onto the embedded finite accessible subspace. Extend the decoder by

$$\mathscr{D}(Y) = \widehat{\mathscr{D}}(P_R Y P_R) + \mathrm{Tr}\left((I - P_R)Y\right)\tau_C,$$

with the compression identified with *R*. This map is CPTP and agrees on proxy outputs. Postprocessing contraction adds *b*, outside the square root. The constant decoder supplies *a_d*. The direct certificate follows by applying the Choi bound to the specified decoded proxy channel, followed by the same lift.

This is the certificate of Hawk04-A/B with its normalization and continuum-lift distinction retained [CheungHawk04A,CheungHawk04B]. The dimension factor multiplies the normalized Choi discrepancy, not an allowance already expressed in diamond distance.

Theorem. Fix *C*, common separable output spaces *R_{*}, B_{*}*, and factorwise embeddings of a sequence *V_{ℓ}* . Suppose the embedded isometries *∇_{ℓ}* converge in operator norm to *V_{*}*, with

$$\left\| \overline{V}_\ell - V_* \right\|_\infty \le \beta_\ell, \qquad \beta_\ell \to 0.$$

Then *V_{*}* is an isometry, and

$$d_\diamond(\overline{\mathscr{N}}_\ell, \mathscr{N}_*) \le \min\{1, \beta_\ell\}, \qquad d_\diamond(\overline{\mathscr{E}}_\ell, \mathscr{E}_*) \le \min\{1, \beta_\ell\}.$$

The recovery and leakage diagnostics differ from their limit values by at most *min {1, β_{ℓ}}*.

Proof. Inner products pass to the limit, so *V_{*}^{†}V_{*} = I_C*. Apply Lemma , including invariance under the local embeddings.

The theorem requires a complete isometry comparison. Convergence of selected outgoing occupation numbers is not its premise.

Growing input dimensions require a separate contract

Let $C_n \simeq \mathbb{C}^{d_n}$, with $d_n \to \infty$.

Definition. A growing-code study may use either:

• **Fixed-subcode convergence.** For each fixed finite code *C^{(k)}*, supply embeddings *J_{n, k}: C^{(k)} → C_n* and compare *V_nJ_{n, k}* on that fixed domain, with compatible output identifications.

• **Whole-code certificates.** For each *n*, test the target *id_C_n* separately, reporting its own complete proxy allowance *b_n* and Choi discrepancy *e_n*. Neither convention silently equates different identity channels.

Corollary. For whole-code finite-proxy certificates,

$$\varepsilon_{\mathrm{rec}}(\mathscr{N}_n) \le \min\{a_{d_n}, \sqrt{2 d_n e_n} + b_n\}.$$

Thus *b_n → 0* and *d_ne_n → 0* are sufficient for these upper certificates to vanish. The assertion *e_n → 0* alone is insufficient to make this particular certificate vanish.

Proof. Apply Theorem for each *n*.

The last sentence concerns the certificate, not a necessary condition for every successful recovery method. For infinite-dimensional input spaces, energy-constrained norms provide a different comparison framework [Shirokov]; no unrestricted infinite-input diamond limit is inferred here.

A model-specific finite-band regulator

This section supplies one concrete realization of the approximation allowance. It reconstructs the finite-band

regulator used in Hawk04-A/B [CheungHawk04A,CheungHawk04B]. It is independent of the thermal-pair calibration in the next section.

Let *H_S* be a bounded self-adjoint operator on a finite source space. An optional quartic realization is

$$H_S = \sum_{i<j<k<l} J_{ijkl}\chi_i\chi_j\chi_k\chi_l + B_N I, \qquad B_N = \sum_{i<j<k<l} |J_{ijkl}|,$$

where *{χ_i, χ_j} = 2δ_ijI*. Then *0 ≤ H_S ≤ 2B_NI*.

Fix *0 < μ < Λ < ∞*, and set

$$\mathfrak{h} = L^2([\mu,\Lambda], \mathrm{d}\omega; \mathbb{C}^r), \qquad (hf)(\omega) = \omega f(\omega).$$

On *ℱ_s(ℌ)*, let *H_B = dΓ(h)*. For fixed self-adjoint source operators *A_a*, real *g_a*, and *f_a ∈ ℌ*, define

$$H = H_S + H_B + \sum_a g_a A_a \otimes \Phi(f_a), \qquad \Phi(f) = a(f) + a^\dagger(f).$$

Annihilation is conjugate-linear in *f*. Put

$$G = \sum_a |g_a| \|A_a\| \|f_a\|, \qquad G_\omega = \sum_a |g_a| \|A_a\| \left\| h^{-1/2} f_a \right\|.$$

The bath is initially vacuum, and all finite source/carrier preparation is given by one fixed code isometry. Choose a finite collected packet space *ꞅ = span {p_1, …, p_q}*.

Lemma. The Hamiltonian is self-adjoint on the free bath domain and generates a complete unitary group. If *H_S ≥ 0*, it has the form lower bound

$$H \geq \frac{1}{2} H_B - 2G_\omega^2 I.$$

Proof. The field estimates give

$$\|a(f)\psi\| \leq \left\| h^{-1/2} f \right\| \left\| H_B^{1/2} \psi \right\|,$$

and

$$\left\| a^\dagger(f)\psi \right\|^2 = \|a(f)\psi\|^2 + \|f\|^2 \|\psi\|^2.$$

Hence the interaction obeys

$$\|V_{\mathrm{int}}\psi\| \leq 2G_\omega \left\| H_B^{1/2}\psi \right\| + G\|\psi\|.$$

The square-root term has arbitrarily small relative operator bound with respect to *H_B*. Kato–Rellich therefore gives self-adjointness [Kato]; these are standard field-operator estimates [HaslerHerbst]. For normalized form-domain vectors,

$$|\langle V_{\mathrm{int}} \rangle| \leq 2G_\omega \sqrt{\langle H_B \rangle} \leq \tfrac{1}{2}\langle H_B \rangle + 2G_\omega^2.$$

The spectral theorem supplies the unitary group.

No ground-state or scattering theorem is imported from the field references.

Let *P_m* project onto bath occupation at most *m*, and put

$$Z_m(x) = \sum_{\ell=m+1}^{\infty} \frac{x^\ell}{\sqrt{\ell!}}.$$

Theorem. For fixed cutoffs, packets, and *T < ∞*, there is a finite prepared proxy isometry *𝒱_t*, embedded factorwise into the complete output, such that uniformly for *0 ≤ t ≤ T*,

$$d_\diamond(\mathscr{V}_t, \widehat{\mathscr{V}}_t) \leq b_{\mathrm{reg}}(T; m, \Delta) := \min\left\{1, 2Z_m(2GT) + \frac{Tm\Delta}{2}\right\}.$$

The same bound applies to both reduced channels and their diagnostic differences.

For *T > 0* and *0 < b < 1*, sufficient choices are

$$m \geq 1, \quad m+1 \geq 4\mathrm{e}(2GT)^2, \quad m \geq 2 + \log_2(1/b), \quad \Delta \leq \frac{b}{Tm}.$$

They give $b_reg \le b$.

Proof. In the interaction picture,

$$V_I(s) = \sum_a g_a A_a(s) \otimes \Phi(e^{ish} f_a).$$

On sectors with occupation at most k,

$$\|V_I(s)\psi\| \le 2G\sqrt{k+1}\,\|\psi\|.$$

An ℓ-fold product from vacuum has norm at most $(2G)^{\ell}\sqrt{(\ell!)}$. After integration over the ordered simplex, its Dyson term is bounded by $(2Gt)^{\ell}/\sqrt{(\ell!)}$. The series converges also after multiplication by any fixed polynomial in occupation. On these prepared vectors it solves the Schrödinger equation in the required domains and equals the self-adjoint evolution.

The compressed Hamiltonian P_mHP_m has identical Dyson terms through order m. Inserted projections are contractions, so the two prepared isometries differ by at most $2Z_m(2GT)$.

Partition the frequency band into bins of width at most Δ. Replacing frequency multiplication by bin midpoints gives $\|h - \hbar\| \le \Delta/2$. On the occupation-cutoff space,

$$\left\| P_m \mathrm{d}\Gamma(h - \hat{\tilde{h}}) P_m \right\| \le m\Delta/2.$$

Both compressed Hamiltonians are bounded, so Duhamel's formula adds at most $Tm\Delta/2$.

The active one-particle space

$$\mathfrak{f} = \mathrm{span}\,\{1_{I_b} f_a, 1_{I_b} p_j : a, b, j\}$$

is finite dimensional, invariant under $\hbar$, and contains all couplings and collected packets. Its orthogonal complement remains vacuum. The total-occupation-cutoff space is finite dimensional and embeds into

$$\mathscr{F}_{\mathrm{s_}} \le m(\mathfrak{r}) \otimes \mathscr{F}_{\mathrm{s_}} \le m(\mathfrak{f} \cap \mathfrak{r}^{\perp}).$$

This is an embedding, not an equality of the total cutoff with a tensor product. Source and carrier stay in the inaccessible factor. Lemma converts the isometry allowance to .

Finally let $x = 2GT$. The factorial estimate yields

$$\frac{x^{m+1}}{\sqrt{(m+1)!}} \le \left(\frac{\mathrm{e}x^2}{m+1}\right)^{(m+1)/2} \le 2^{-(m+1)}.$$

Later term ratios are at most $1/2$, hence $Z_m(x) \le 2^{-m}$. The occupation and mesh terms are each at most $b/2$ under .

The theorem refines occupation and frequency proxies on a bounded time interval. It does not remove μ, Λ, and its constants are not uniform under their removal. It proves no collapse geometry, no local field-to-memory receiver, and no asymptotic completeness theorem. A different Hamiltonian requires a different analytical comparison.

Thermal calibration is not a complete emission channel

An independent thermal-pair cutoff

Use $\hbar = k_B = 1$ in this subsection. For one oscillator frequency $\omega > 0$, temperature $T > 0$, and $\lambda = e^{-\omega/T}$, define

$$|\Theta_\lambda\rangle = \sqrt{1-\lambda} \sum_{r=0}^{\infty} \lambda^{r/2} |r\rangle_T |r\rangle_P.$$

Its T-marginal is

$$\tau_\lambda = (1-\lambda) \sum_{r=0}^{\infty} \lambda^r \mathrm{kb}\, rr, \qquad \mathrm{Tr}\,(N\tau_\lambda) = \frac{\lambda}{1-\lambda}.$$

The normalized cutoff preparation is

$$|\Theta_{\lambda,m}\rangle = \sqrt{\frac{1-\lambda}{1-\lambda^{m+1}}} \sum_{r=0}^{m} \lambda^{r/2} |r\rangle_T |r\rangle_P.$$

Proposition. These preparations satisfy

$$d_{\mathrm{tr}}(|\Theta_\lambda\rangle\langle\Theta_\lambda|, |\Theta_{\lambda,m}\rangle\langle\Theta_{\lambda,m}|) = \lambda^{(m+1)/2},$$

$$d_{\mathrm{tr}}(\tau_\lambda, \tau_{\lambda,m}) = \lambda^{m+1},$$

$$\mathrm{Tr}\,(N\tau_\lambda) - \mathrm{Tr}\,(N\tau_{\lambda,m}) = \frac{(m+1)\lambda^{m+1}}{1-\lambda^{m+1}}.$$

For L independent pairs with individual cutoffs m_{ℓ}, the complete pure-state distance is

$$\sqrt{1 - \prod_{\ell=1}^{L} (1-\lambda_\ell^{m_\ell+1})} \le \sqrt{\sum_{\ell=1}^{L} \lambda_\ell^{m_\ell+1}}.$$

Appending these fixed preparations to the same logical isometry gives the same complete-channel allowance, independently of the code dimension.

Proof. The squared overlap of the complete pair preparations is $1 - \lambda^{m+1}$, giving the pure-state distance. The discarded geometric tail has weight λ^{m+1}; the normalized retained distribution gives the marginal distance. Differentiating the finite geometric sum gives

$$\mathrm{Tr}\,(N\tau_{\lambda,m}) = \frac{\lambda}{1-\lambda} - \frac{(m+1)\lambda^{m+1}}{1-\lambda^{m+1}}.$$

Product overlaps multiply, and $1 - \prod_{\ell}(1-t_{\ell}) \le \sum_{\ell} t_{\ell}$. For any referenced logical input, the preparation difference is a tensor product with a trace-one logical output, so its trace norm is precisely the preparation trace norm.

This is a state-preparation certificate. It does not establish that a finite matrix is an exact truncation of a Bogoliubov propagator.

The Bogoliubov bridge has separate hypotheses

A proposed free-field bridge must provide:

• the spacetime and state preparation, field algebra, mode normalizations, and incoming and outgoing access;

• a canonical transformation with its complete partner and purifier accounting;

• an implementability statement in the chosen representation;

• a fixed-code complete-channel comparison to its finite proxy;

• moment or flux control for the unbounded observables used in energy predictions. Unitary implementability of bosonic canonical transformations is a nontrivial issue in infinite dimensions [BruneauDerezinski]. It is not supplied by choosing a lattice.

Nor can finite matrices satisfy the exact oscillator relation $[a, a^{\dagger}] = I$, because the trace of a finite commutator is zero. Finite truncation therefore requires a stated approximation and cannot be described as an exact finite representation of the full CCR.

Even trace-distance convergence does not alone control an unbounded number observable. For example,

$$\rho_n = (1-n^{-1})\mathrm{kb}\,00 + n^{-1}|n^2\rangle\langle n^2|$$

converges in trace distance to vacuum, but $\mathrm{Tr}\,(N\rho_n) = n$. Equation supplies moment control for its particular geometric preparation; a collapse model must supply its own.

Equal thermal and entropy diagnostics, opposite recovery

Proposition. Choose a finite state σ_K, a purification $|\chi\rangle_{KP}$, and $L \simeq C$. Let $R = C_R \otimes K$, $B = P \otimes L$, and define

$$V_{\mathrm{return}}|\psi\rangle = |\psi\rangle_{C_R}|\chi\rangle_{KP}|0\rangle_L, \qquad V_{\mathrm{retain}}|\psi\rangle = |0\rangle_{C_R}|\chi\rangle_{KP}|\psi\rangle_L.$$

Then

$$\mathcal{N}_{\mathrm{return}}(X) = X \otimes \sigma_K, \qquad \mathcal{N}_{\mathrm{retain}}(X) = \mathrm{Tr}\,(X)\mathrm{kb}\,00 \otimes \sigma_K.$$

For every pure code input, their accessible spectra and entropies coincide. For $H_R = I_C_R \otimes H_K$, all accessible energy-measurement distributions coincide for every input. Nevertheless,

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}_{\mathrm{return}}) = 0, \qquad \varepsilon_{\mathrm{rec}}(\mathcal{N}_{\mathrm{retain}}) = a_d.$$

Taking σ_K Gibbs makes these selected energy statistics thermal.

Proof. Both maps preserve inner products, and partial traces give the channels. For pure inputs, both outputs have the nonzero spectrum of σ_K. Every spectral effect of H_R acts identically on the logical factor. Tracing K decodes the return map; the retain map is a replacement, so Lemma applies.

The same separation is reported in the supplied En07-C excerpts and Hawk04-C [CheungEn07C,CheungHawk04C]. A complete channel, once specified, fixes its complementary information up to the standard environmental equivalence. The missing information here is the complete channel specification, not a failure of dilation theory [Stinespring].

Stage 3: energy, erasure cost, and a conditional mass law

Three independent ledgers

The ledgers have different defining data:

• **Complete energy.** Supply the Hamiltonians and complete dynamics. Conservation means $U^{\dagger}H_{tot}U = H_{tot}$, or, for an energy-intertwining isometry, $H_{out}V = VH_{in}$. Unitarity alone does not imply either statement.

• **Physical erasure/reset cost.** Supply the memory, input class, reservoirs, work storage, controls, initial correlations, heat/work conventions, and resource-restoration conditions.

• **Residual leakage.** Compute the dimensionless complete-channel diagnostic δ_{full}. It is not an energy, stress tensor, area, or bit count. This separation is central to En07-A/B/C [CheungEn07A,CheungEn07B,CheungEn07C].

Proposition. Equality of a selected visible channel does not determine a unique heat or work ledger, even after a conserving spectator Hamiltonian is fixed.

Proof. Append independent qubits W, L with $H_W = H_L = \varepsilon kb\,11$, $\varepsilon > 0$. Prepare W excited and L in $\gamma_L = diag\,(1 - p, p)$, $0 < p < 1$. Doing nothing and applying SWAP both conserve $H_W + H_L$ and leave the selected visible channel unchanged. The first has designated reservoir energy change zero, while the second has $Q_L = \varepsilon(1 - p) > 0$, supplied by the decrease of W's energy. The consumed resource differs. These are not cyclic erasure protocols unless the spectator resources are restored.

Proposition. Let a memory and finite reservoir start in $\rho_M \otimes \gamma_L$, with $\gamma_L = e^{-\beta H_L}/Tr\, e^{-\beta H_L}$, $\beta > 0$. After a specified unitary, define

$$Q_L = \mathrm{Tr}\,(H_L(\rho_L' - \gamma_L)), \qquad \Delta S_M = S(\rho_M) - S(\rho_M').$$

Using natural logarithms,

$$\beta Q_L = \Delta S_M + I(M : L)_{\rho'} + D(\rho_L' \| \gamma_L) \geq \Delta S_M.$$

Proof. Unitary entropy preservation and the initial product state give

$$S(\rho_L') - S(\gamma_L) = \Delta S_M + I(M : L)_{\rho'}.$$

Using $\log \gamma_L = -\beta H_L - \log Z$ gives

$$D(\rho_L' \| \gamma_L) = \beta Q_L - [S(\rho_L') - S(\gamma_L)].$$

Combine the identities and use nonnegativity.

Equation is the thermal entropy-balance identity associated with the additional implementation assumptions, not a cost derived from $dropBit \circ Enter = Tr$ [Landauer,ReebWolf].

The luminosity and clock are additional inputs

Use $G = \hbar = c = k_B = 1$ only for the physical matching formulas in this subsection and the mass estimates below.

Thus *T_H(M) = 1/(8πM)*.

Assumption. On *M ∈ [M_c, M_0]*, with *M_c > 0*, a proposed model supplies a time normalization, outgoing flux observable, and power law

$$P(M) = \sum_{\alpha} \int_0^{\infty} \frac{\mathrm{d}\omega \qquad \omega\, \Gamma_{\alpha}(M,\omega)}{2\pi \exp\left(\omega / T_{\mathrm{H}}(M)\right) - \eta_{\alpha}}.$$

Here *η_{α} = 1* for bosons and *−1* for fermions; degeneracies are included in *α*. The integrals and sum are finite. For the cubic-scaling specialization, the species are massless and fixed, and

$$\Gamma_{\alpha}(M,\omega) = \gamma_{\alpha}(M\omega).$$

The mass balance is additionally supplied as *M = −P(M)*, or with a declared forcing error.

The flux normalization and absorption factors are part of a physical emission calculation, as in Hawking and Page [Hawking1975,PageEmission]. A temperature-labelled finite state does not supply them.

Theorem. Under Assumption in its fixed massless-species specialization,

$$P(M) = \frac{\kappa}{M^2},$$

where

$$\kappa = \sum_{\alpha} \int_0^{\infty} \frac{\mathrm{d}x \qquad x\, \gamma_{\alpha}(x)}{2\pi \exp\left(8\pi x\right) - \eta_{\alpha}}.$$

If *κ > 0* and *M = −κ/M^2*, then until the validity cutoff,

$$M(t)^3 = M_0^3 - 3\kappa t, \qquad t_{\mathrm{c}} = \frac{M_0^3 - M_{\mathrm{c}}^3}{3\kappa}.$$

For fixed *M_c* and *κ*, this cutoff lifetime has cubic large-*M_0* scaling.

Proof. Set *x = Mω* in . The measure times the energy factor contributes *M^{−2}*, and the thermal denominator becomes *exp (8πx) − η_{α}*. Then *d(M^3)/dt = 3M^2M = −3κ*. Integrate and solve for the first crossing of *M_c*.

Massive thresholds, changing species, rotation, charge, incoming radiation, or time-dependent validity assumptions can change this specialization. No coefficient or Planck-scale extrapolation is predicted by the finite Enter interface.

Moreover, temperature matching without a clock is insufficient even abstractly: traversing the same sequence of thermal states with a rescaled time parameter preserves those state observations while changing its duration. A lifetime requires a flux-and-time law.

The theorem predicts time to a positive cutoff under its explicit luminosity assumptions. It does not predict the remaining lifetime, final burst, or remnant behavior below that cutoff. Finite termination of an imposed schedule supplies none of those missing physical conclusions.

Stability, adiabaticity, and finite-step errors

Theorem. Suppose an absolutely continuous trajectory remains in *[M_c, M_0]* until its first cutoff crossing and satisfies

$$(1-\rho)\frac{\kappa}{M^2} \le -\dot{M} \le (1+\rho)\frac{\kappa}{M^2}, \qquad 0 \le \rho < 1.$$

Then it reaches the cutoff and

$$\frac{M_0^3 - M_{\mathrm{c}}^3}{3\kappa(1+\rho)} \le t_{\mathrm{c}} \le \frac{M_0^3 - M_{\mathrm{c}}^3}{3\kappa(1-\rho)}.$$

More generally, if

$$\dot{M} = -P(M), \qquad \dot{\widetilde{M}} = -P(\widetilde{M}) + r(t),$$

and *P* is Lipschitz with constant *L* on a common validity interval, then, while both trajectories stay there,

$$|\widetilde{M}(t) - M(t)| \le e^{Lt}|\widetilde{M}(0) - M(0)| + \int_0^t e^{L(t-s)}|r(s)|\,\mathrm{d}s.$$

Proof. Multiply the first differential inequalities by *3M^2*. The derivative of *M^3* lies between *−3κ(1 + ρ)* and *−3κ(1 − ρ)*. Integration gives finite cutoff crossing and the time bracket. For the second claim, the difference satisfies the integral inequality

$$|z(t)| \le |z(0)| + \int_0^t L|z(s)|\,\mathrm{d}s + \int_0^t |r(s)|\,\mathrm{d}s.$$

Gronwall's inequality gives .

This locally proves the type of classical forcing stability requested in the Hawk02-A plan, without importing an unprovided numerical forcing estimate [CheungHawk02A].

Proposition. Let *f(M) = −κ/M^2*, with *|f| ≤ K* and Lipschitz constant *L* on a common positive-mass domain. For step size *h*, let *M_{j + 1} = M_j + hf(M_j)*. If the exact solution, numerical iterates, and connecting intervals remain in that domain, then

$$|M_j - M(jh)| \le \frac{Kh}{2}(e^{Ljh} - 1).$$

One may take *K = κ/M_c^2* and *L = 2κ/M_c^3* on *[M_c, M_0]*.

Separately, suppose a finite proxy Hamiltonian obeys

$$\|H(M) - H(M')\| \le K_H|M - M'|, \qquad |\dot{M}| \le v.$$

Replacing the actual mass path over one step by its initial value changes the complete unitary channel by at most *K_Hvh^2/2* in half-diamond distance. Multiple compatible step allowances add by telescoping.

Proof. The exact solution's local Euler residual is at most

$$\int_0^h L|M(t_j + s) - M(t_j)|\,\mathrm{d}s \le LKh^2/2.$$

Thus the global error obeys *e_{j + 1} ≤ (1 + Lh)e_j + LKh^2/2*. Sum the geometric series and use *(1 + Lh)^j ≤ e^Ljh*.

Duhamel's formula bounds the propagator difference by

$$\int_0^h \|H(M(t_j + s)) - H(M(t_j))\|\,\mathrm{d}s \le K_H v h^2/2.$$

Apply the isometry-to-channel estimate and CPTP contraction at each matched interface.

For *T(M) ∝ M^{−1}*, a dimensionless slow-variation indicator is

$$\chi_{\mathrm{ad}} = \tau_{\mathrm{corr}}\frac{|\dot{T}|}{T} = \tau_{\mathrm{corr}}\frac{|\dot{M}|}{M}.$$

Small *χ_ad* is not a channel certificate by itself. The final statement of Proposition requires a Hamiltonian comparison constant, which may depend on the regulator.

Feedback and logical-input dependence

A prescribed external *M(t)* gives an input-independent family of quantum channels. A measured record controlling later emissions gives a different adaptive hybrid channel, which is CPTP when the complete instruction tree is specified.

A rule that computes *M* from an unknown input state's expectation and then selects a state-dependent propagator is not automatically a linear quantum channel. It must be replaced by an explicit quantum or classical control implementation, or restricted to an input-independent energy sector. Neither fresh inputs nor input-dependent preparations may be silently inserted while testing one fixed code.

Mass updates must use specified energy increments, not the number of classical bits produced by Enter. An apparatus can generate a record without that record count being an outgoing gravitational energy flux.

Stage 4: information flow, entropy tests, and Page-time limits

The two directions of approximate constancy

Proposition. If $d_{\diamond}(\mathcal{N}, \mathcal{T}_{\omega}) \le \eta$, then

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}) \ge \max\{0, a_d - \eta\}.$$

Near constancy of the complete complement instead gives a recovery upper certificate through or its finite-proxy lift.

Proof. Every decoded replacement is still a replacement. Postprocessing contraction and the triangle inequality give

$$d_{\diamond}(\mathcal{D}\mathcal{N}, \mathrm{id}_C)\mathrm{ged}_{\diamond}(\mathcal{D}\mathcal{T}_{\omega}, \mathrm{id}_C) - \eta \ge a_d - \eta.$$

Take the decoder infimum.

This is the directional distinction retained in En07-B/C and Hawk04-A/B/C [CheungEn07B,CheungEn07C,CheungHawk04A, CheungHawk04B,CheungHawk04C]. A radiation state near maximally mixed for one input is not the uniform referenced channel hypothesis in .

Lemma. If $x = d_{tr}(\mathcal{E}(\rho_0), \mathcal{E}(\rho_1))$, then $\delta_{full}(\mathcal{E}) \ge x/2$.

Proof. For every replacement state σ, the triangle inequality gives

$$x\mathrm{led}_{\mathrm{tr}}(\mathcal{E}(\rho_0), \sigma) + d_{\mathrm{tr}}(\sigma, \mathcal{E}(\rho_1)) \le 2d_{\diamond}(\mathcal{E}, \mathcal{T}_{\sigma}).$$

Take the infimum.

A whole-code dimension constraint

Proposition. For a finite accessible output of dimension $r \le d$,

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}) \ge 1 - \frac{r}{d}.$$

Exact recovery with constant residual rank s additionally requires $r \ge ds$.

Proof. The Bell-input state on $Q \otimes R$ is a mixture of pure vectors of Schmidt rank at most r. Applying each decoder Kraus operator cannot increase that rank. A normalized vector of Schmidt rank at most r has squared Bell overlap at most r/d, by Cauchy–Schwarz on its Schmidt coefficients. Convexity and the Bell-projector test prove . The exact statement is Theorem .

If C is the entire unknown n-qubit input and only k qubits are accessible, this gives $\varepsilon_{rec} \ge 1 - 2^{k - n}$. A claim of accurate whole-code recovery after half the qubits must therefore change its input target, output dimension, or access assumptions. A small diary inserted into a previously correlated system is not this whole-code task [HaydenPreskill].

Entropy turnover is not a decoder transition

For a pure input in the flagged model, the radiation state has orthogonal occupied and vacuum blocks, so

$$S(R_j) = h_2(q_j), \qquad h_2(q) = -q\log q - (1-q)\log(1-q).$$

Its entropy is maximal at $q_j = 1/2$, where the optimal recovery error is $a_{d/2}$, not zero. Also $h_2(q) = h_2(1-q)$, although the respective recovery errors qa_d and $(1-q)a_d$ differ.

More generally, Proposition can be applied at successive times with the same prepared ancilla spectrum $\sigma_{K,t}$. It gives identical selected entropy histories while one logical channel returns the code and the other retains it. Producing a turnover in $\sigma_{K,t}$ does not repair the hidden-code model. This is the entropy/recovery separation motivating the supplied Hawk01-B comparison [CheungHawk01B].

If radiation is cumulatively retained and never reinteracts, more access cannot worsen unrestricted recovery: whenever $\mathcal{N}_s = P_{t \to s}\mathcal{N}_t$ with $P_{t \to s}$ CPTP,

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}_t) \le \varepsilon_{\mathrm{rec}}(\mathcal{N}_s).$$

Indeed a decoder at s can be composed with the restriction. This availability statement is not a streaming-memory

or efficient execution theorem [CheungEn02C].

A joint-pair consistency test

Theorem. Let R be earlier radiation and b, c a new outgoing qubit and partner. If

$$S(bc) \le s, \qquad S(c) \ge \log 2 - u,$$

then

$$S(Rb) - S(R) \ge \log 2 - s - u.$$

If $d_{tr}(\rho_{bc}, |\Phi^{+}\rangle\langle\Phi^{+}|) \le e \le 1/2$, then

$$S(Rb) - S(R) \ge \log 2 - 2h_2(e) - e\log 3.$$

If successive steps have no reinteraction of earlier radiation and each right-hand side is at least $\gamma > 0$, then $S(R_N) \ge S(R_0) + N\gamma$.

Proof. Weak monotonicity, a consequence of strong subadditivity [LiebRuskai], gives

$$S(Rb) + S(bc) \ge S(R) + S(c).$$

For the trace-distance specialization, finite-dimensional entropy continuity gives $S(bc) \le h_2(e) + e\log 3$, and partial-trace contraction gives $S(c) \ge \log 2 - h_2(e)$ [Audenaert]. The bounds are monotone on the stated range. The final assertion is telescoping under the no-reinteraction premise.

This reconstructs the supplied Hawk02-A pair test [CheungHawk02A]. It is consistent with the role of specified joint pair structure in small-corrections arguments [Mathur]. Its trace-distance parameter is not silently identified with another paper's amplitude parameter. A thermal-looking outgoing marginal does not establish its joint-pair premise.

A nonzero-coupling negative control

The following check uses the commuting four-Majorana member of the family in Hawk04-A/B [CheungHawk04A,CheungHawk04B]. Take

$$H_S = J(I - S_1S_2), \qquad A = (S_1 + S_2)/2, \qquad J > 0, \quad g \ne 0,$$

where $S_j = 2c_j^{\dagger}c_j - I$. Insert the diary into the source states $s_0 = |00\rangle$, $s_1 = |11\rangle$. They have source energy zero and A-eigenvalues $-1, +1$. Couple to one finite-band scalar field with $f(\omega) = 1/\sqrt{(2\omega)}$. Define

$$\eta_i(t) = g a_i \frac{e^{-i\omega t} - 1}{\omega} f(\omega), \qquad a_0 = -1, \quad a_1 = 1.$$

For a collected packet projection $P_{\mathfrak{r}}$, let

$$c_R(t) = \exp\left[-\frac{1}{2}\|P_{\mathfrak{r}}(\eta_0 - \eta_1)\|^2\right], \qquad z_R = \sqrt{1 - c_R^2}.$$

Proposition. The radiation channel is

$$\mathcal{N}_t(\rho) = \rho_{00}|\eta_{0,R}\rangle\langle\eta_{0,R}| + \rho_{11}|\eta_{1,R}\rangle\langle\eta_{1,R}|.$$

It has

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}_t) = \frac{3 - z_R(t)}{4} \ge \frac{1}{2}, \qquad \delta_{\mathrm{full}}(\mathscr{E}_t) \ge \frac{1}{2},$$

although the complete output is exactly recoverable. Its energy ledger is

$$E_S(t) = 0, \qquad E_B(t) = g^2 \int_{\mu}^{\Lambda} \frac{1 - \cos(\omega t)}{\omega^2}\, d\omega, \qquad E_I(t) = -E_B(t).$$

Proof. For a fixed source eigenvalue, Weyl displacement completes the square in $H_B + g a_i \Phi(f)$. Vacuum evolves, up to a common phase, to the coherent amplitude η_i. Tracing the orthogonal source labels gives the displayed measure-and-prepare channel.

The two collected pure states have trace distance z_R. Any decoder's Bell overlap is at most $(1 + z_R)/4$: its two logical diagonal adjoint effects define a binary discrimination measurement, whose equal-prior success probability

is at most *(1 + z_R)/2*, obtained from the positive part of the state difference. The Bell test gives error at least *(3 − z_R)/4*. Parity interchanges the two states, so an optimal discrimination measurement may be symmetrized to have equal conditional errors. Measuring it and preparing the inferred basis state yields

$$z_R \Delta_Z + (1 - z_R)\mathcal{T}_{I_2/2}.$$

Its Pauli identity weight is *(1 + z_R)/4*; orthogonal Pauli Bell outputs give the matching error.

The residual retains orthogonal source labels on the two basis inputs, so Lemma gives leakage at least *1/2*. Complete preservation follows from Lemma and Theorem . The coherent bath energy is *‖h^{1/2}η_i‖^2*, giving the integral. The initial total energy is zero, and conservation gives *E_I = −E_B*.

This is a negative control, not a generic scrambling claim. Bath energy here is a dressing response of an initially undressed preparation, not a derived gravitational mass loss.

Stage 5: endpoint obligations and order of limits

Definition. For a fixed finite code *C*, an endpoint package supplies:

• A complete pure preparation *P_0: C → H_tot*, including all fixed purifiers.

• Input-independent complete unitary evolution *U_t*, with a justified time domain.

• Output partitions *R_t ⊗ B_t*, factorwise isometries *j_t: R_t → R_{*}*, *k_t: B_t → B_{*}*, and an endpoint map *V_{*}* such that

$$\overline{V}_t = (j_t \otimes k_t)V_t, \qquad \left\|\overline{V}_t - V_*\right\|_\infty \le v_t, \qquad v_t \to 0.$$

The embeddings preserve access roles.

• Either exact constancy of *𝒞_{*} = Tr_{R_{*}} Ad_{V_{*}}*, or an explicit approximate endpoint certificate:

• finite endpoint outputs with *d_{◇}(𝒞_{*}, 𝒯_{σ_{*}}) ≤ ζ*; or

• a finite endpoint proxy with complete allowance *b_{*}* and residual Choi discrepancy *e_{*}*. For a physical application, these data must be justified in one compatible model, with a receiver realizing the declared access.

This is a sufficient contract, not a necessary characterization of all possible endpoints. It is the four-obligation distinction in the supplied En07-B/C excerpts, with the finite-proxy option added to avoid an unproved infinite-output approximate-decoder assertion [CheungEn07B,CheungEn07C].

Theorem. Under Definition , *V_{*}* is an isometry and the embedded complete and reduced channels are within *min {1, v_t}* of their endpoint channels.

If *𝒞_{*}* is exactly constant, one fixed endpoint decoder *𝒟_{*}: R_{*} → C* gives local finite-time decoders *𝒟_t = 𝒟_{*}Ad_j_t* with

$$d_\diamond(\mathcal{D}_t \mathcal{N}_t, \mathrm{id}_C) \le \min\{1, v_t\}.$$

For a finite endpoint with allowance *ζ*, the same construction has error at most

$$\min\{1, v_t + \sqrt{2\zeta}\}.$$

For the finite-proxy endpoint option, a fixed extended proxy decoder has error at most

$$\min\{1, v_t + b_* + \sqrt{2de_*}\}.$$

A specified endpoint proxy decoder instead gives *min {1, v_t + b_{*} + dr_{dec, *}}*.

If the exact complete residual is pure, the endpoint radiation is an isometric encoding. If it is mixed, the syndrome normal form applies.

Proof. Operator-norm convergence preserves *V_{*}^{†}V_{*} = I_C*. Lemma gives channel convergence. For exact constancy, Theorem supplies *𝒟_{*}*. For a finite approximate endpoint, and finite attainment supply an endpoint decoder. For the proxy option, use Theorem . In each case, postprocessing contraction adds at most *v_t*. The decoder *𝒟_t* acts only on *R_t*. Rank one makes the syndrome space one dimensional.

The endpoint statement continues the access-preserving construction in Hawk04-A/B/C [CheungHawk04A,CheungHawk04B,CheungHawk04C]. A comparison unitary that swaps a still-inaccessible

logical factor into the radiation output is not an access-preserving identification.

Proposition. For finite input and separable normal outputs, $\delta_{full}(\mathscr{E}) = 0$ implies that $\mathscr{E}$ is a replacement channel.

Proof. Choose states σ_n with $d_{\diamond}(\mathscr{E}, \mathscr{T}_{\sigma_n}) \to 0$. Evaluation on I_C/d gives $\sigma_n \to \mathscr{E}(I_C/d)$ in trace distance. Replacement-channel distance equals the corresponding state distance. The triangle inequality gives $\mathscr{E} = \mathscr{T}_{\mathscr{E}(I_C/d)}$.

Information-free is not physically absent

A constant complement contains no logical input dependence in the declared full-code task. It need not be one dimensional, pure, empty, or zero energy. Conversely, a zero-energy source can have a degenerate memory retaining the entire code:

$$V|\psi\rangle = |0\rangle_R|\mathrm{vac}\rangle_S|\psi\rangle_M, \qquad H_S|\mathrm{vac}\rangle = 0.$$

Radiation is constant, while the complete residual recovers the input.

The explicit endpoint maps in the supplied Hawk01-B excerpts transfer all pending code and records to radiation and leave a fixed residual [CheungHawk01B]. That rule is an additional construction. Calling a source ``evaporated'' does not supply it.

A complete classical-output channel on $d \geq 2$ cannot have a one-dimensional pure-dilation environment: otherwise it would itself be an isometry and have an exact decoder, contradicting Lemma . Thus an empty list of live program names does not prove an empty complete physical complement.

Recurrence and a diagonal limiting schedule

Proposition. For a finite-dimensional autonomous Hamiltonian, its complete prepared channel returns arbitrarily close to its initial channel at arbitrarily late times. If accessible radiation is initially fixed vacuum, then

$$\limsup_{t \to \infty} \varepsilon_{\mathrm{rec}}(\widehat{\mathscr{N}_t}) = a_d.$$

Proof. The finitely many spectral phases have simultaneous approximate returns to one. Equivalently, compactness of the phase torus and the group property produce arbitrarily late returns; a periodic orbit uses multiples of its period. The propagator approaches the identity along such a sequence. Lemma gives channel recurrence and recovery stability. The initial channel is a replacement with error a_d, and every channel has recovery error at most a_d using the constant decoder.

This is the fixed-proxy obstruction in Hawk04-A/B/C [CheungHawk04A,CheungHawk04B,CheungHawk04C].

If a continuum endpoint is approximated at increasing times, one must choose a diagonal regulator schedule satisfying, for example,

$$b_{\mathrm{reg}}(T_\ell; m_\ell, \Delta_\ell) \to 0$$

on the actual increasing windows T_{ℓ}, together with the endpoint access comparison and decoupling estimates. For the sufficient selection in , occupation grows at least quadratically with GT_{ℓ}, and the mesh must shrink accordingly. A finite-time theorem does not justify interchanging the regulator and late-time limits.

The combined conditional certification theorem

Definition. A proposed extrapolation supplies:

• A fixed logical target, complete preparation, dynamics, field content, regulators, validity domain, and all inaccessible outputs.

• A justified accessible radiation interface and receiver, with compatible complete comparisons for preparation, evolution, collection, and implementation.

• A finite proxy and a rigorously feasible residual state or quantum-return decoder. The transfer law is labelled *derived* or *additionally assumed*.

• For a mass prediction, a specified flux, time normalization, mass balance, cutoff, and quantitative power-error control.

• For an endpoint prediction, the same-model package in Definition .

• For an experimental claim, a detector instrument, input class, efficiency, background model, and statistical confidence convention. No scalar entropy, metric, or energy error is added to a diamond budget without a conversion theorem.

For compatible complete-channel comparisons let

$$b_{\mathrm{tot}} = \min\left\{1, b_{\mathrm{prep}} + \sum_j b_j + b_{\mathrm{reg}} + b_{\mathrm{collection}} + b_{\mathrm{implementation}}\right\}.$$

Every allowance must refer to the same complete output accounting after specified identifications; omitted sectors already counted in the complement are not charged twice.

Theorem. Under the applicable parts of Definition :

• Compatible stepwise complete comparisons give actual-to-proxy complete-channel distance at most *b_tot*.

• A proxy residual Choi discrepancy *e* gives

$$\delta_{\mathrm{full}}(\mathscr{E}) \le \min\{1, de + b_{\mathrm{tot}}\}, \qquad \varepsilon_{\mathrm{rec}}(\mathscr{N}) \le \min\{a_d, \sqrt{2de} + b_{\mathrm{tot}}\}.$$

A specified decoder gives error at most *min {1, dr_dec + b_tot}*.

• An independently proved accessible replacement comparison gives the lower bound *max {0, a_d − η}*.

• A matched inverse-square luminosity with relative power error $\rho < 1$ gives the cutoff-lifetime bracket . It does not determine below-cutoff behavior.

• The endpoint package gives the fixed-decoder bounds of Theorem .

• Any prescribed subsequent detector channel inherits the decoded channel allowance by CPTP contraction.

• Residual Enter followed by Drop leaves the already supplied accessible channel unchanged and adds no final-burst conclusion. The channel, mass, and endpoint conclusions follow from separate hypotheses. None supplies a missing hypothesis of another.

Proof. Telescoping matched CPTP compositions and contraction prove (i). Theorem proves (ii); Proposition proves (iii). Theorems and prove (iv). Theorem proves (v). Postprocessing contraction proves (vi), and Proposition proves (vii). The hypotheses are explicitly partitioned by their mathematical objects, so the final scope assertion makes no additional physical identification.

The theorem is a conditional certification architecture in the spirit of the supplied Hawk02-A acceptance interface [CheungHawk02A]. It is not a report that its gravitational hypotheses have been discharged.

What the numerical and experimental tests can establish

For a specified decoded channel, let its Bell overlap be

$$F = \langle \Phi_d | J_{\mathrm{n}}(\widehat{\mathscr{D}\mathscr{N}}) | \Phi_d \rangle.$$

A Bell-projector test always gives the error lower witness $1 - F$. A validated lower bound $F \ge F_0$ also gives

$$r_{\mathrm{dec}} \le \sqrt{1 - F_0},$$

by the pure-target fidelity–trace-distance inequality [FuchsVanDeGraaf]. Hence the dimension-explicit upper certificate is

$$d_{\diamond}(\mathscr{D}\mathscr{N}, \mathrm{id}_C) \le \min\{1, d\sqrt{1 - F_0} + b_{\mathrm{tot}}\}.$$

High Bell overlap is therefore useful, but its conversion to a uniform diamond certificate must retain the dimension factor or use a sharper independently proved channel estimate.

Repeated calibrated preparations and measurements can estimate such quantities. They are not one-run tomography of an unknown diary. Statistical bounds must state their confidence level; numerical feasibility and physical preparation/receiver errors remain separate.

Proposition. If two predicted channels are within *b* in half-diamond distance, then for every allowed referenced preparation and subsequent detector instrument, their classical output distributions have total variation distance at most *b*.

Proof. Compose the preparation, channels, and detector. CPTP contraction bounds the final trace distance by *b*.

For diagonal classical states this is total variation distance.

A failure of a matched empirical prediction challenges at least one member of the physical, approximation, receiver, or statistical package. It does not falsify the identity *dropBit* ∘ *Enter* = *Tr*. Conversely, agreement with selected counts does not establish a complete-channel certificate.

Prediction targets and a reproducible next-calculation plan

Corrected targets

begingroup 1.12 Conditional prediction targets. An experimental signature is a test of the specified combined model, not a consequence of typing alone.

toprule Target Proved finite ingredient Additional input Defensible test

midrule endfirsthead toprule Target Proved finite ingredient Additional input Defensible test

midrule endhead bottomrule endfoot

Cutoff lifetime Sequential energy calibration; ODE and perturbation bounds Flux/time normalization, greybody scaling, species, validity cutoff Luminosity or lifetime data compared with the stated bracket, including detector and population assumptions

Entropy turnover Selected entropy examples and the joint-pair consistency test A specified state history and microscopic entropy calculation Entropy measurements in a calibrated analogue; not automatically a decoder test

Late quantum recovery Constant-complement criterion and finite Choi/decoder certificates Complete residual audit, coherent collection, feasible quantum decoder Referenced quantum-return tests with a dimension-explicit confidence certificate

Terminal energetic emission A terminal instruction is well typed; readout-and-drop leaves the old radiation channel unchanged Below-cutoff dynamics, charged energy budget, spectral and detector model A specified energetic distribution or correlation; no burst follows from Enter rate

Information-free endpoint Exact or approximate endpoint complement certificate Same-model preparation, dynamics, access, and endpoint limit Recovery or complete residual-independence tests in a controlled model

Absence of physical remnants No finite channel identity proves it Microscopic endpoint ontology, stable-sector audit, production and detection model Statistical exclusion of a stated remnant class, not universal absence from a null observation

endgroup

Six calculations, with completion criteria

• **Analytic baseline before simulation.** Implement the *n = 8–12* SWAP schedule only after specifying whether the target is the whole *n*-qubit input or a smaller diary. Compare against Theorem . Independently implement the occupied-vacuum pulse model and compare against *q_ja_d*. These are different channels and must not share an unqualified formula.

• **A finite-band microscopic proxy.** Archive *H_S*, *A_a*, *g_a*, *f_a*, cutoffs, code preparation, packets, *T*, *m*, *Δ*, and the exact finite active space. Choose regulators before evaluating the recovery certificate. Report the proved *b_reg* , not merely a matrix size.

• **A separate field bridge.** If studying a collapsing shell, archive the geometry, canonical transformation, representation, partner modes, and complete fixed-code comparison theorem. Do not substitute from a different Hamiltonian. Thermal-pair preparation may first be checked using Proposition ; it is not the collapse calculation.

• **Mass matching and stability.** Declare whether the mass path is imposed or derived. Supply the outgoing energy observable and emission clock. Validate the power law on a positive-mass domain, report the cutoff lifetime bracket, and retain all forcing, discretization, switching, and interaction-energy terms.

• **Information and negative controls.** Compute every logical matrix unit of both proxy channels or exploit a proved symmetry reduction. Validate a residual state or CPTP decoder. Include the commuting control of Proposition , the spectral separation, and a record-only test. A designed *q_j* drop is labelled designed, not discovered.

• **Release the full certificate package.** Archive channel data, feasible Choi matrices, validated positivity, trace-preservation and trace-norm bounds, arithmetic allowances, quantum-return programs, separate terminal-readout programs, complete apparatus outputs, and all physical matching assumptions. For endpoint claims, include access-preserving identifications and the order of limits.

No numerical run is reported in this manuscript. For a whole n-qubit input, there are $d^2 = 4^n$ logical matrix units. Generic complete-channel certification is therefore not automatically tractable at $n = 12$. Analytic symmetries, smaller fixed diaries, or validated structured representations must be justified rather than assumed.

For a finite residual Choi state J, a feasible trace-norm upper certificate can be produced by

$$\begin{aligned} &\text{minimize} && \tfrac{1}{2}\mathrm{Tr}\,(X_+ + X_-), \\ &\text{subject to} && J - I_Q/d \otimes \sigma = X_+ - X_-, \\ & && X_\pm \succeq 0, \quad \sigma \succeq 0, \quad \mathrm{Tr}\,\sigma = 1. \end{aligned}$$

A feasible tuple certifies an upper bound on e, not the exact diamond norm. A finite decoder's unnormalized Choi operator must satisfy

$$J_D \succeq 0, \qquad \mathrm{Tr}_C\, J_D = I_R^{\wedge}.$$

Floating-point objective values alone certify neither condition.

Standing constraints and conclusion

The completed plan yields local channel theorems, explicit calibration models, controlled finite-time proxy comparisons, and conditional mass and endpoint implications. Its conclusions retain the following limits.

• No metric, event horizon, area theorem, stress-energy solution, backreaction geometry, or gravitational field-to-memory receiver is constructed.

• Hawking temperature and greybody matching are external physical data. A selected Gibbs marginal is not a complete logical channel.

• The cubic formula is a time-to-cutoff consequence of an imposed or independently derived inverse-square luminosity. It is not a Planck-scale endpoint theorem.

• No universal Page-time transition is obtained. Whole-code recovery, small-diary recovery, entropy turnover, and reference decoupling have different hypotheses.

• Readout-and-drop is operational forgetting, not annihilation. Complete apparatus and discarded-record outputs remain accounted for.

• An information-free residual need not be physically absent. Absence of detectable objects requires a specified empirical exclusion model.

• A mathematical decoder and an abstract qubit dictionary are not an efficient implementation, a finite-gate compilation certificate, or a coherent physical receiver.

• The finite-band regulator is not transferred to another Hamiltonian or to cutoff removal by analogy. Fixed finite autonomous proxies recur.

• No catalogue attributed to OpenAI is used as a premise. The separate documentary statuses and analytic-admission programmes mentioned by the companion manuscripts are not updated by this paper.

The mathematical extrapolation is therefore:

$$\text{boxed}\quad \begin{aligned} &\text{complete physical comparison + finite feasible certificate} \Rightarrow \text{referenced channel bound}, \\ &\text{specified luminosity and mass balance} \Rightarrow \text{conditional cutoff-lifetime bound}, \\ &\text{access-preserving endpoint and decoupling} \Rightarrow \text{endpoint quantum recovery}. \end{aligned}$$

None of these arrows supplies the missing premise of another.

The Enter philosophy is useful precisely because it keeps the additional obligations visible. The resulting programme can generate quantitative conditional predictions and calibrated analogue tests. A claim about actual black-hole evaporation requires a same-model physical construction, not a renaming of the finite outputs.

Declarations

Verification and reproducibility. The evidence consists of the analytical arguments printed here. Compilation, numerical optimization, proof-assistant checking, independent technical review, and acceptance of this manuscript are not asserted. No experimental or simulated data are reported.

Attribution. Established dilation, correction, no-hiding, entropy, and information–disturbance results are attributed to their external literature. Cheung companion manuscripts are version-qualified comparisons. Local proofs do not establish priority for the underlying standard principles.

Authorship and submission metadata. The author field follows the supplied Hawk04-A/B/C sources. The responsible submitter should confirm authorship, affiliations, journal-specific declarations, and archival availability of unpublished comparisons before submission.

appendix

Debugging and provenance ledger

Mathematical repairs incorporated into the paper

toprule Draft shortcut Replacement used here

midrule endfirsthead toprule Draft shortcut Replacement used here

midrule endhead bottomrule endfoot

Decoder allowance $2de + b$ The general complementary-Choi allowance is $\sqrt{(2de)} + b$. A specified decoded Choi state instead gives $dr_dec + b$.

Diamond convergence with changing d_n Fixed-subcode comparisons or separate whole-code certificates are required. Whole-code upper certificates retain d_ne_n.

One regulator for scalar reservoir and collapse Bogoliubov map The finite-band theorem is proved for its printed Hamiltonian. A field bridge has separate representation, implementability, access, and approximation obligations.

Exact finite oscillator algebra Finite matrices cannot obey the full CCR. Normalized finite preparations and finite-channel comparisons are used instead.

Bits emitted imply mass decrease Energy and mass updates use declared flux observables and a clock. Record count alone is not an energy ledger.

$T \propto M\^\{-1\}$ implies $M\^3$ lifetime Fixed species, greybody scaling, flux normalization, and mass balance are also required; the theorem stops at $M_c > 0$.

Half the quanta imply recovery of the whole code The target and accessible dimension are fixed first. The SWAP calibration and dimension lower bound obstruct that universal claim.

Entropy turnover implies small complete leakage Explicit equal-entropy/unequal-recovery channels disprove this inference. A separate complete-complement certificate is required.

Final Enter creates an energetic burst Readout-and-drop leaves prior radiation unchanged. Any burst requires a new energetic dynamics and detector model.

Thermalization makes the environment disappear Input independence, thermal state preparation, and physical absence are different claims. Purifiers remain in complete accounting.

Null remnant detection proves no remnants Only a stated class can be statistically excluded under a production, efficiency, and background model.

Finite termination proves irreversible evaporation Finite syntax terminates; finite autonomous proxies recur. An endpoint needs its own limit and access construction.

Source boundary

The complete Hawk04-A version 1, Hawk04-B version 2, and Hawk04-C version 1 sources were supplied for this development. The Hawk01-B, Hawk02-A/B, and En07-A/B/C comparisons additionally use the version-qualified excerpts supplied with the request. Their full unseen proof packages are not asserted to have been inspected.

The Philosophy essay was consulted at its stated companion address. Cheung's journal article has an independently identifiable publisher record. That article's status does not assign publication dates or journal acceptance to the companion manuscripts.

The retrieval labels, duplicated catalogue extracts, and automated ``further development'' paragraphs in the supplied material are not mathematical premises. OpenAI catalogue material is excluded.

toprule Source Comparison retained

midrule endfirsthead toprule Source Comparison retained

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Hawk04-A@1 Complete restoration, Schmidt decoder, finite-band regulator, dimension-explicit certificate, negative control, and access-preserving endpoint.

Hawk04-B@2 Typed destructive readout, quantum return, apparatus accounting, finite proxies, and recurrence.

Hawk04-C@1 Six-row operational analogy, SWAP schedule, complete-complement criterion, three ledgers, and standing non-identifications.

Hawk02-A@1 Supplied stability, pair-entropy, mass-forcing, and conditional physical acceptance interfaces. Corresponding statements used here are proved locally.

Hawk02-B@2 Supplied single-cell pulse survival and optimal flagged recovery benchmark. The model and formulas are reconstructed locally.

Hawk01-B@1 Supplied entropy/recovery separation and explicit information-free endpoint transfer. No endpoint is inferred without that added rule.

En07-A/B/C@1 Supplied three-ledger separation, endpoint obligations, KSW direction, and selected spectral/entropy counterexamples.

En02-C@1 Sequential reference accounting and the distinction between available information and executable recovery under a resource budget.

A compact archival checklist

A submission reporting a future positive physical or numerical result should additionally archive:

• the exact code target and reference convention;

• complete preparation, all purifiers, source/carrier ordering, and every inaccessible factor;

• the frozen Hamiltonian or canonical transformation and its existence/implementability proof;

• the accessible algebra and physical receiver comparison;

• regulator choices and complete-channel allowances on the actual time domain;

• all logical matrix-unit data or a proved equivalent reduction;

• feasible residual and decoder Choi operators with validated positivity, normalization, and arithmetic bounds;

• the quantum-return program and any distinct terminal observation;

• full energy, interaction, switching, heat/work, and resource-return ledgers;

• luminosity matching, clock, mass cutoff, and perturbation bounds;

• endpoint embeddings, complete residual audit, and order of limits;

• detector likelihoods, efficiencies, backgrounds, and confidence statements. No successful branch, selected population test, disorder-averaged channel, or entropy plot substitutes for this package.

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