Hawk04-C Finite Enter Analogues of the Hawking Narrative: Access, Complete Complements, and Destructive Readout

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Abstract

We give a finite-dimensional, reference-preserving formulation of six operational analogies associated with the Hawking narrative. The mathematical objects are a complete isometry, a declared accessible factor, its complete residual complement, destructive computational-basis readout, and classical discard. We prove complete-output restoration, the exact constant-complement criterion for full-code recovery, finite-program soundness, the sharp classical-record recovery obstruction, and the equality of readout-and-discard with partial trace. The presentation corrects several possible overstatements: an arbitrary isometry need not relocate the whole code into its residual; a nonconstant complement need not independently recover that code; unobserved residual readout cannot improve the existing accessible channel; and consumption of all named quantum registers does not prove that a complete physical complement has disappeared. A worked register-transfer schedule and a corrected quantum-computer narrative illustrate the analogies. Quantitative recovery and endpoint statements are separated from the exact results and explicitly identify their additional hypotheses. No metric, event horizon, Hawking spectrum, mass-loss dynamics, gravitational evaporation endpoint, or gravitational Page curve is derived. The contribution is an operational synthesis and separation argument, not a solution of black-hole evaporation.

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Background and the Enter discipline

The external physical narrative

The classical and semiclassical statements being compared have different hypotheses. Classical black-hole mechanics concerns a spacetime metric and its dynamics; an event horizon is a causal-geometric object, not an access declaration on a register. The area theorem belongs to that classical setting, with its geometric and energy assumptions. It is not a theorem about the number of accessible qubits [Hawking1971,BardeenCarterHawking1973].

Hawking's particle-creation calculation concerns quantum fields on a classical collapse background. For an uncharged, nonrotating Schwarzschild black hole, its temperature, in SI units, is

$$T_{\mathrm{H}} = \frac{\hbar c^{3}}{8\pi G M k_{\mathrm{B}}}.$$

For bosonic modes, the corresponding occupation formula has a thermal denominator and an absorption, or greybody, factor:

$$\langle n_{\omega\ell m}\rangle = \frac{\Gamma_{\omega\ell m}}{\exp\left(\hbar\omega/(k_{\mathrm{B}}T_{\mathrm{H}})\right)-1}.$$

Here ω is angular frequency. These are external semiclassical statements [Hawking1975]. In particular, a heuristic picture of escaping particle pairs is not used below as a microscopic circuit construction.

The further questions of information loss, outgoing quantum recovery, and the final regime require more than an occupation formula. Hawking's information-loss proposal and Page's unitary-radiation comparison address this broader setting under their respective assumptions [Hawking1976,Page1993]. The phrase ``final explosive burst'' recalls a physical extrapolation discussed in Hawking's early work; it is not treated here as an established endpoint theorem [Hawking1974,Hawking1975].

What the Philosophy of Enter contributes

Cheung's *Philosophy of Enter* supplies an interface discipline: destructive readout consumes the measured quantum name and returns a classical record; quantum recovery instead returns a quantum code from specified access. It also separates complete energy, erasure cost, and residual leakage, and refuses to identify an analogy with its physical interpretation [CheungPhilosophy]. We implement that discipline by specifying, for every claim:

• the input code and its retained external references;

• the accessible output and everything excluded from it;

• whether the output is quantum, classical, or hybrid;

• the channel identity or error bound actually established.

The finite readout convention below is *defined*, not derived from measurement-uniqueness assumptions. Cheung's journal article provides background on the Enter calculus and its operational characterization of measurement [CheungEnter]. Neither that citation nor a citation to Manuscript I reconstructs the full hypotheses of its characterization in the present model.

Relationship to the companion manuscripts

The principal comparisons are Hawk04-A and Hawk04-B, which separate complete preservation from restricted quantum recovery, and En07-A, En07-B, En07-C, and En04-A, which make the readout, complement, endpoint, and response distinctions explicit [CheungHawk04A,CheungHawk04B,CheungEn07A, CheungEn07B,CheungEn07C,CheungEn04A].

All exact channel statements needed for the six rows are proved locally. The companion citations identify related formulations, not imported model-specific Hamiltonians, field preparations, or gravitational dictionaries. The underlying exact error-correction, dilation, and no-hiding principles are established quantum information theory [KnillLaflamme,Stinespring,BraunsteinPati]; no priority claim is made for them.

The source material supplied for Hawk04-A@1 and En07-B@1 includes version-pinned excerpts rather than an independently inspected complete proof package. Appendix records the source boundary. No external OpenAI-family proof is a premise of this manuscript.

Finite carriers, complete outputs, and access

All Hilbert spaces in the mathematical results below are finite dimensional. Write $\mathscr{L}(H)$ for the linear operators on a Hilbert space H and $\mathscr{D}(H)$ for its density operators. A channel is a completely positive trace-preserving linear map. Let

$$C \simeq \mathbb{C}^d, \qquad d \geq 2,$$

be the entire logical code whose identity channel is the recovery target. The symbol Q denotes an arbitrary untouched external reference.

Definition. A complete interface consists of an isometry

$$V : C \to R \otimes B, \qquad V^\dagger V = I_C,$$

and a declaration that R is accessible while B is the complete inaccessible output. Its channels are

$$\mathcal{V}(X) = VXV^\dagger, \qquad \mathcal{M}(X) = \mathrm{Tr}_B\,(VXV^\dagger), \qquad \mathcal{E}(X) = \mathrm{Tr}_R\,(VXV^\dagger).$$

The complement B contains every unreturned quantum output in the declared model, including preparation purifiers, retained source registers, uncollected modes, and inaccessible apparatus registers. The independent reference Q is not part of B.

The listed components of B are an accounting requirement. They are not an assertion that an unregulated field theory admits the finite tensor factorization . A finite field-to-memory preparation, if physically claimed, must be supplied separately.

A sufficient mathematical preparation hypothesis is an isometry $J: C \to H_in$, a fixed normalized pure ancillary state $|\Omega\rangle_A$, and a unitary U on the complete prepared space. After a fixed output identification,

$$V|\psi\rangle = U(J|\psi\rangle \otimes |\Omega\rangle_A).$$

A mixed fixed ancillary preparation can be purified only by retaining its purifier in the complete accounting. An input-dependent preparation must itself be specified as a code-to-complete-system map; it is not covered by assuming a fixed ancilla.

Finite spaces can be embedded in qubit registers with unused basis states. This observation is a mathematical encoding convention, not a physical field receiver or a gate-compilation certificate.

Definition. For $\sigma \in \mathcal{D}(B)$, define the replacement channel

$$\mathcal{T}_\sigma(X) = \mathrm{Tr}\,(X)\sigma.$$

Use half-normalized trace and diamond distances:

$$d_{\mathrm{tr}}(\rho,\eta) = \frac{1}{2}\|\rho-\eta\|_1, \qquad d_\diamond(\Phi,\Psi) = \frac{1}{2}\|\Phi-\Psi\|_\diamond.$$

For differences of channels with input C, the latter distance is the supremum of the referenced output trace distance over input states on $Q \otimes C$; a reference of dimension d suffices. Set

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}) = \inf_{\mathcal{D}\,:\,\mathcal{L}(R)\,\to\,\mathcal{L}(C)\ \mathrm{CPTP}} d_\diamond(\mathcal{D}\mathcal{N}, \mathrm{id}_C),$$
$$\delta_{\mathrm{full}}(\mathcal{E}) = \inf_{\sigma\,\in\,\mathcal{D}(B)} d_\diamond(\mathcal{E}, \mathcal{T}_\sigma).$$

A decoder is deterministic, and its output is quantum. A normalized successful branch alone is not a decoder in .

The finite-reference reduction follows from Schmidt decomposition for pure inputs and convexity for mixed inputs. Equality of channels on all matrix units consequently preserves every external reference.

Local preservation and exact full-code recovery

Theorem. For , choose any $\tau_C \in \mathcal{D}(C)$ and put $P = VV\^\{\dagger\}$. The map

$$\mathcal{L}_V(Y) = V^\dagger Y V + \mathrm{Tr}\,((I_{RB}-P)Y)\tau_C$$

is CPTP and satisfies

$$\mathcal{L}_V\mathcal{V} = \mathrm{id}_C.$$

For every operator X on $Q \otimes C$,

$$\left\|(I_Q\otimes V)X(I_Q\otimes V^\dagger)\right\|_1 = \|X\|_1.$$

Thus all logical matrix units and all referenced state distinctions are preserved on the complete output. For a maximally entangled code–reference input, the complete output has

$$I(Q:RB) = 2\log_2 d.$$

Proof. The first term of is a completely positive compression. The second is a positive-effect measurement followed by preparation of τ_C, hence is completely positive. Their traces sum to

$$\mathrm{Tr}\,(PY) + \mathrm{Tr}\,((I-P)Y) = \mathrm{Tr}\,Y.$$

On $Y = VXV\^\{\dagger\}$, the second term vanishes and the first returns X. This proves the left-inverse identity on all

operators.

An isometry preserves all nonzero singular values of an operator under left and right embedding. The same is true of $I_Q \otimes V$, proving . The statement about matrix units also follows directly from the left inverse.

Finally, let

$$|\Phi_d\rangle = \frac{1}{\sqrt{d}} \sum_{i=0}^{d-1} |i\rangle_Q |i\rangle_C.$$

The state $(I_Q \otimes V)|\Phi_d\rangle$ is pure with d equal Schmidt coefficients. Its Q and RB entropies are both $\log_2 d$, and its joint entropy vanishes.

The domain of $\mathscr{L}_V$ is $R \otimes B$. It is a complete-access quantum decoder, not generally a decoder on R. Its off-range completion does not make it a unitary inverse on all of $R \otimes B$. Compare the corresponding restore maps in [CheungHawk04A] and [CheungEn07A].

Fix a code basis and let

$$E_{ij} = |i\rangle\langle j|, \qquad J_{\mathrm{n}}(\Phi) = (\mathrm{id}_Q \otimes \Phi)(|\Phi_d\rangle\langle\Phi_d|) = \frac{1}{d} \sum_{i,j} E_{ij}^{Q} \otimes \Phi(E_{ij}).$$

Theorem. For the complete interface , the following are equivalent:

• A CPTP decoder satisfies $\mathscr{D}\mathscr{N} = \mathrm{id}_C$.

• $\mathscr{E} = \mathscr{T}_{\sigma_B}$ for one $\sigma_B \in \mathscr{D}(B)$.

• $\mathscr{E}(E_{ij}) = \delta_{ij}\sigma_B$ for every matrix unit.

• $J_n(\mathscr{E}) = I_Q/d \otimes \sigma_B$.

• For every $O \in \mathscr{L}(B)$,

$$V^{\dagger}(I_R \otimes O)V = \mathrm{Tr}\,(O\sigma_B) I_C.$$

• For a complete Kraus family $\{K_a\}$ of $\mathscr{N}$, every $K_a^{\dagger} K_b$ is a scalar multiple of I_C. If $r = \mathrm{rank}\,\sigma_B$, there are a space $K \simeq \mathbb{C}^r$, a state σ_K, and an isometry $A: C \otimes K \to R$ such that

$$\mathscr{N}(X) = A(X \otimes \sigma_K)A^{\dagger}.$$

An exact decoder on *all* of $\mathscr{L}(R)$ is

$$\mathscr{D}(Y) = \mathrm{Tr}_K\,(A^{\dagger} Y A) + \mathrm{Tr}\,((I_R - AA^{\dagger})Y)\tau_C.$$

In particular,

$$\dim R \geq d\, r.$$

Every recovery identity in the theorem retains arbitrary references.

Proof. Matrix units span $\mathscr{L}(C)$, so (ii) and (iii) are equivalent. The displayed Choi expansion proves equivalence with (iv). Trace duality gives

$$\mathrm{Tr}\,(O\mathscr{E}(X)) = \mathrm{Tr}\,(V^{\dagger}(I_R \otimes O)VX),$$

which proves equivalence between (ii) and (v).

In a basis $\{|a\rangle\}$ of B, choose $K_a = (I_R \otimes \langle a|)V$. Then

$$[\mathscr{E}(X)]_{ab} = \mathrm{Tr}\,(K_a X K_b^{\dagger}) = \mathrm{Tr}\,(K_b^{\dagger} K_a X).$$

Consequently (ii) implies (vi). Conversely, if these products are scalar, evaluation at I_C/d shows that their scalar entries form exactly the density operator $\mathscr{E}(I_C/d)$. The same equality for every X proves (ii).

Assume (ii). Diagonalize the nonzero part of the residual state:

$$\sigma_B = \sum_{\alpha=1}^{r} p_\alpha |b_\alpha\rangle\langle b_\alpha|, \qquad p_\alpha > 0, \quad \sum_{\alpha} p_\alpha = 1.$$

Each $V|i\rangle$ is supported on $R \otimes \mathrm{supp}\,\sigma_B$. Indeed, its squared component in the complementary residual support

equals the expectation of that support projection in σ_B, which is zero. We may therefore write

$$V|i\rangle = \sum_{\alpha=1}^{r} \sqrt{p_\alpha}|u_{i\alpha}\rangle_R|b_\alpha\rangle_B.$$

Comparing the coefficient of $|b_\{\alpha\}\rangle\langle b_\{\beta\}|$ in $\mathscr{E}(E_ij) = \delta_ij\sigma_B$ gives

$$\langle u_{j\beta} \mid u_{i\alpha}\rangle = \delta_{ij}\delta_{\alpha\beta}.$$

These are dr orthonormal vectors in R, proving . Define

$$A(|i\rangle|\alpha\rangle) = |u_{i\alpha}\rangle, \qquad \sigma_K = \sum_\alpha p_\alpha|\alpha\rangle\langle\alpha|.$$

The orthogonality makes A an isometry, and tracing B proves .

Both terms in are completely positive. Their traces sum to $Tr\ Y$. Substitution of gives

$$\mathscr{D}_{\mathscr{N}}(X) = \mathrm{Tr}_K(X \otimes \sigma_K) = X.$$

Thus (ii) implies (i), with a decoder defined off the encoded range as well as on it.

Conversely, suppose (i). Take Kraus operators $D_\{\gamma\}$ of the decoder and form the isometry

$$L : R \to C \otimes G, \qquad L|r\rangle = \sum_\gamma D_\gamma|r\rangle \otimes |\gamma\rangle.$$

This is the finite Kraus form of a Stinespring dilation [Stinespring]. Set $Z = (L \otimes I_B)V$. On basis input $|i\rangle$, its decoded marginal is pure, so

$$Z|i\rangle = |i\rangle \otimes |\xi_i\rangle_{GB}$$

for a normalized vector ξ_i. Exact recovery of E_ij requires

$$\langle \xi_j \mid \xi_i\rangle = 1.$$

Hence all the ξ_i are one vector ξ. Because L acts only on R, tracing its outputs cannot change the B-marginal. It follows that

$$\mathscr{E}(E_{ij}) = \delta_{ij}\mathrm{Tr}_G\,|\xi\rangle\langle\xi|.$$

This is (iii).

The identities hold on all matrix units and therefore on every reference extension by linearity.

This is the full-input-space constant-complement form of exact Knill–Laflamme correction [KnillLaflamme]. Compare the local formulations in [CheungHawk04A] and [CheungEn07C].

Corollary. The complete residual B exactly recovers C if and only if $\mathscr{N}$ is a replacement channel. The accessible factor R exactly recovers C if and only if $\mathscr{E}$ is a replacement channel. For $d \geq 2$, the two factors cannot both independently recover the entire code, and cannot both have constant channels.

Proof. Apply Theorem with the output factors interchanged. A replacement channel cannot recover two distinct code states. If one factor recovers, the other is constant; this rules out both independent recovery and simultaneous constancy.

Corollary. If the complete complement is nonconstant, then $\varepsilon_rec(\mathscr{N}) > 0$.

Proof. Decoder Choi matrices are positive, have a fixed partial trace, and have fixed total trace. They form a compact set in finite dimensions. The decoder error is continuous, so its infimum is attained. If that minimum were zero, Theorem would force a constant complement.

Remark. Theorem concerns identity recovery of the *entire* code. For the smaller target Tr_S on $C_L \otimes C_S$, the channel $\mathscr{N} = Tr_S$ returns C_L exactly while a complementary channel returns the generally nonconstant state on C_S. Full-code constancy must not be imposed as a necessary condition for that different task.

Row 1: the collapse analogy

Description. A finite ``star'' means a prepared code inside a declared finite register model. Applying preserves the code on the complete output. Calling the resulting inaccessible register a black hole is an analogy.

There is an important additional distinction: *complete preservation is not complete relocation into the residual*. The latter requires a separate channel property.

Definition. An interface has residual-only relocation when a CPTP map on B recovers the entire code, including its reference correlations.

By Corollary , residual-only relocation is equivalent to $\mathcal{N}$ being a replacement channel. It is not a consequence of $V^{\dagger}V = I_C$ alone.

Example. Let $R \simeq B \simeq C$ and define

$$V_{\text{hidden}}|\psi\rangle = |0\rangle_R|\psi\rangle_B, \qquad V_{\text{visible}}|\psi\rangle = |\psi\rangle_R|0\rangle_B.$$

Both are complete isometries. The first has residual-only relocation and constant accessible output. The second has exact accessible recovery and constant residual output. For qubit registers, the first can be obtained by a specified SWAP with a blank register.

Proof. For *V_hidden*,

$$\mathcal{N}_{\text{hidden}}(X) = \mathrm{Tr}\,(X)\mathrm{kb}\,00, \qquad \mathscr{E}_{\text{hidden}}(X) = X.$$

The formulas are interchanged for *V_visible*. They establish the assertions on all operators and references.

Thus a mathematically correct version of the first row is: After an independently supplied finite preparation, a complete isometry preserves the code globally. A chosen relocation isometry can place its full recovery interface in the residual. The code is what is encoded. Fixed source and apparatus ancillas are not additional unknown logical inputs unless the declared code has been enlarged to include them.

No event horizon, singularity, area theorem, or surface of no return is derived from the Enter readout convention or from Theorem . No energy-to-geometry implication is asserted.

Row 2: the event-horizon analogy

Description. The finite analogue is the declared access split $R \mid B$. This specifies the decoder's domain. It is not a geometric surface.

Lemma. Let $\Lambda: \mathscr{L}(B) \to \mathscr{L}(B^{'})$ be CPTP. For every state on $Q \otimes R \otimes B$,

$$\mathrm{Tr}_{B'}\left((\mathrm{id}_{QR} \otimes \Lambda)(\rho)\right) = \mathrm{Tr}_B \rho.$$

Proof. If $\{L_j\}$ is a Kraus family of Λ, then $\sum_j L_j^{\dagger}L_j = I_B$. For every observable A on $Q \otimes R$,

$$\begin{aligned}\mathrm{Tr}\left[(A \otimes I_{B'})(\mathrm{id}_{QR} \otimes \Lambda)(\rho)\right] &= \sum_j \mathrm{Tr}\left[(A \otimes L_j^{\dagger}L_j)\rho\right] \\ &= \mathrm{Tr}\left[(A \otimes I_B)\rho\right].\end{aligned}$$

All expectations on QR agree, proving .

Corollary. If a code has constant accessible channel at a fixed cut, operations confined to B, with no outputs delivered across that cut, cannot make that accessible channel input dependent.

Proof. Apply Lemma to each local operation and compose.

Consequently, a narrative in which qubits ``leak across the cut'' must specify an actual transfer interface or an access change. For example, an isometry

$$W : B \to S \otimes B^{'}$$

may return S to the observer, changing the next accessible factor to $R \otimes S$. Alternatively, explicitly granting the observer access to the old B changes the recovery problem without changing the global state.

For *V_hidden*, the observer with R alone cannot recover. An observer with $R \otimes B$ can recover by Theorem . These are different access contracts, not conflicting conclusions.

The split $R \mid B$ constructs neither a null surface nor a metric. It carries no area monotonicity, Lorentzian causal structure, or gravitational trapping statement. The no-signalling identity is not the classical area theorem [BardeenCarterHawking1973].

Row 3: the radiation and readout analogy

Description. Before measurement, ``radiation'' denotes a declared accessible quantum factor. After destructive Enter, the measured part is replaced at the observable interface by a classical record. Those two interfaces must not be conflated.

Definition. Let S be a measured qubit register and T the surviving quantum register. For computational strings z, put

$$M_z = \langle z|_S \otimes I_T.$$

Destructive Enter has the hybrid channel

$$\mathcal{M}_S(X) = \sum_z \mathrm{kb}\, zz_K \otimes M_z X M_z^\dagger.$$

The output names are K and T, not S. The algebra on K is the diagonal classical algebra.

Classical discard acts on a hybrid block-diagonal operator by

$$\mathrm{dropBit}_K\Big(\sum_z \mathrm{kb}\, zz_K \otimes X_z\Big) = \sum_z X_z.$$

The notation extends to a finite bit string. It returns no register K.

These formulas instantiate the finite convention documented in Hawk04-B and En07-A [CheungHawk04B] [CheungEn07A]. They are independently specified here, rather than imported as a uniqueness theorem.

Proposition. The map is CPTP. For a state ρ, outcome z has probability

$$p_z = \mathrm{Tr}\,(M_z \rho M_z^\dagger).$$

If $p_z > 0$, its surviving state is $M_z\rho M_z\^{\{\dagger\}}/p_z$. For every untouched reference, the same formulas apply with M_z tensored with the reference identity. Moreover,

$$\mathrm{dropBit}_K \mathcal{M}_S(X) = \mathrm{Tr}_S X.$$

Proof. Kraus operators for the hybrid map are $|z\rangle_K \otimes M_z$. They satisfy

$$\sum_z M_z^\dagger M_z = I_{ST},$$

which proves complete positivity and trace preservation. Taking the trace of one classical block gives p_z. Positive-probability normalization gives the conditional state. No normalized conditional state is required for a zero-probability branch.

Finally,

$$\sum_z (\langle z|_S \otimes I_T) X (|z\rangle_S \otimes I_T) = \mathrm{Tr}_S X$$

is the basis formula for partial trace. All these identities remain true after tensoring with id_Q.

If $R = S \otimes A$, the post-readout accessible factor is $K \otimes A$, while B remains outside the observer's access. An unconditioned readout on S leaves the marginal on B unchanged. A *conditional* residual can change. For example, reading one half of $(|00\rangle + |11\rangle)/\sqrt{(2)}$ prepares the other half in $|z\rangle$ on branch z, although its average state remains $I/2$.

The instrument does not derive or . Producing bits does not establish an outgoing energy flux. A selected thermal state or occupation law is not a specification of a complete input–output channel or its physical complement. This is the spectrum/interface distinction emphasized in En07-C [CheungEn07C,Hawking1975].

Row 4: the shrinking and mass-loss analogy

A specified sequential transfer interface

Let R_0 be trivial and let $V_0: C \to B_0$ be an isometry. Choose finite emission isometries

$$W_j : B_{j-1} \to S_j \otimes B_j, \qquad j = 1, \ldots, t.$$

Define

$$R_j = S_1 \otimes \cdots \otimes S_j, \qquad V_j = (I_{R_{j-1}} \otimes W_j)V_{j-1}.$$

Every register not returned in *R_j* belongs to the complete *B_j*.

Proposition. Every *V_j* in is an isometry. If subsequent emissions act only on the residual and are not controlled by earlier measurement outcomes, destructive readout of an already emitted *S_j* commutes, as a channel, with those subsequent emissions. Immediate and deferred readout therefore give the same final hybrid channel, including every reference.

Proof. Induction gives

$$V_j^\dagger V_j = V_{j-1}^\dagger (I \otimes W_j^\dagger W_j) V_{j-1} = I_C.$$

Readout on an already emitted factor and emission on the residual act on disjoint tensor factors. Their Kraus operators commute after the fixed output reorderings. Composing the two orders therefore gives identical unnormalized branch maps and hence the same hybrid channel. Tensoring with *id_Q* preserves the identity.

The commuting argument is not a storage statement. Deferred readout can require more live quantum memory than immediate readout. It also does not apply unchanged when the emitter uses classical feed-forward from the earlier records.

General finite adaptive programs

Theorem. Consider a finite acyclic instruction tree built from:

• allocation of specified pure ancillas;

• specified unitary operations;

• destructive Enter;

• classical control and classical discard;

• return of a fixed declared hybrid output type. Every quantum name is either returned or explicitly consumed. The tree denotes a CPTP channel. After refining all histories, there are path maps *K_h* satisfying

$$\sum_h K_h^\dagger K_h = I.$$

Its reported output block *y* is the sum

$$\Phi_y(X) = \sum_{h\,:\,\mathrm{out}(h)\,=\,y} K_h X K_h^\dagger.$$

The denotation retains every reference, and each evaluation branch terminates.

Proof. Pure allocation is an isometry. A specified unitary is trace preserving. Enter has complete measurement bras by ∑_z *M_z^{†}M_z = I*. Classical discard adds blocks, and classical control chooses a complete continuation on each block.

Compose the linear path maps from the root to every leaf. Backward induction on the tree, using completeness at each branch, proves . Grouping leaf histories by their reported output proves . These are completely positive maps whose traces sum to the input trace. No normalized postselection is substituted for their sum.

The same path maps tensored with *I_Q* prove the reference statement. A finite acyclic tree has finite maximum branch length, which proves termination.

This is a semantic theorem. Admitting a specified unitary as a mathematical constant does not synthesize it over a fixed gate library. The distinction between channel correctness and actual storage is also present in the sequential certificates of En02-C [CheungEn02C].

If ``fresh content'' is injected during a schedule, it must either be part of the code declared at the beginning or be a separately specified input. One may not silently change the logical input while comparing recovery of one fixed code.

Three independent ledgers

The three ledgers have different defining data:

• **Complete energy.** A declared total Hamiltonian and all charged systems and controls are needed. For example,

U^{†}H_totU = H_tot implies

$$\mathrm{Tr}\,(H_{\mathrm{tot}}U\rho U^{\dagger}) = \mathrm{Tr}\,(H_{\mathrm{tot}}\rho).$$

No such *H_tot* is supplied merely by a channel name.

• **Erasure or reset cost.** Discard returns no register. A reset instead has output

$$\mathrm{Reset}_M\,(X) = \mathrm{Tr}\,(X)\mathrm{kb}\,00_M.$$

A thermodynamic claim additionally needs a memory implementation, reservoir, initial correlations, controls, and a work or heat convention. Landauer's account and finite-reservoir refinements concern such specified physical packages, not just the equality of two reduced channels [Landauer,ReebWolf].

• **Residual leakage.** Equation measures the input dependence of the complete complementary channel. It is a channel property and need not equal an energy, a bit count, or an erasure cost.

For completeness, a simple witness makes the last ledger operational.

Lemma. If

$$x = d_{\mathrm{tr}}(\mathscr{E}(\rho_0), \mathscr{E}(\rho_1)) > 0,$$

then *δ_full(ℰ) ≥ x/2*. Also *δ_full(ℰ) = 0* if and only if *ℰ* is a replacement channel.

Proof. For any *σ*, the triangle inequality gives

$$x \le d_{\mathrm{tr}}(\mathscr{E}(\rho_0), \sigma) + d_{\mathrm{tr}}(\sigma, \mathscr{E}(\rho_1)) \le 2d_{\diamond}(\mathscr{E}, \mathscr{T}_{\sigma}).$$

Take the infimum. In finite dimensions the density operators *σ* form a compact set, so the infimum defining zero leakage is attained. Zero distance is channel equality.

This is a residual-response test, not a statement that the entire code is recoverable from *B*. Compare the corresponding distinction in En04-A [CheungEn04A].

Neither register count nor a chosen temperature-like label is a mass observable. The isometry condition only requires

$$\dim B_{j-1} \le \dim S_j \dim B_j.$$

It does not force monotone residual dimension. After a physical dilation of readout is included, apparatus outputs may remain in the complete residual even while a scheduled source core becomes smaller.

The finite schedule constructs no black-hole Hamiltonian, continuous backreaction, mass-loss differential equation, or evaporation channel. An externally attached temperature or mass schedule remains an input. Shrinking a named register is not a theorem that a gravitational area shrinks.

Row 5: the final-burst and no-remnant analogy

The additional terminal program

A terminal operation on *B* is not available to an observer restricted to *R*. To discuss it, explicitly grant a privileged controller access to the finite residual and add the program

x := Enter(B);?dropBit(x);?returnQ(R);

If *R* is already hybrid, the return preserves its declared hybrid type. No live *B* wire is silently omitted.

Theorem. For every state on *Q ⊗ R ⊗ B*,

$$(\mathrm{id}_{QR} \otimes \mathrm{dropBit}_K)(\mathrm{id}_{QR} \otimes \mathscr{M}_B)(\rho) = \mathrm{Tr}_B\,\rho,$$

where the identities in the displayed composition act on the untouched factors. Equivalently, as a channel from *R ⊗ B* to *R*,

$$P_{\mathrm{forget}B} = \mathrm{dropBit}_K \circ \mathrm{Enter}_B = \mathrm{Tr}_B.$$

Consequently,

$$P_{\mathrm{forget}B}\mathscr{V} = \mathscr{N}.$$

The radiation-only recovery error is exactly the pre-existing *ε_rec(𝒩)*.

Proof. The left side of is

$$\sum_z (I_{QR} \otimes \langle z|_B)\rho(I_{QR} \otimes |z\rangle_B),$$

which is the defining partial trace. Substitute $\rho = (id_Q \otimes \mathcal{V})(X)$ to obtain with every reference. The optimization defining recovery error depends only on the unchanged channel $\mathcal{N}$, so its value is unchanged.

This reproduces the locally stated identity in Hawk04-B [CheungHawk04B]. Lemma proves the stronger conclusion that *any* trace-preserving residual operation, with all its outputs ignored, leaves the untouched channel unchanged.

A coherent implementation does not annihilate information

Proposition. For a basis $\{|z\rangle\}$ of B, the map

$$W|z\rangle_B = |z\rangle_K|z\rangle_E$$

is an isometry. Tracing E gives the destructive classical readout channel. If discarding K is represented by transferring it to an inaccessible store F, the resulting complete isometry can be written

$$J|z\rangle_B = |z\rangle_E|z\rangle_F.$$

Tracing $E \otimes F$ gives Tr_B, while joint EF access restores the original code on B.

Proof. Distinct basis vectors in either formula have orthogonal images, so $W\^{}\{\dagger\}W = J\^{}\{\dagger\}J = I_B$. For $X = \sum_\{z, w\} X_zwkb\ zw$,

$$\mathrm{Tr}_E\,(WXW^\dagger) = \sum_z X_{zz}\mathrm{kb}\,zz_K.$$

Likewise, tracing both outputs of $JXJ\^{}\{\dagger\}$ gives $\sum_z X_zz = Tr\ X$. For a joint state on QRB, the same calculation gives Tr_B while leaving QR untouched. The complete left inverse follows from Theorem , applied to J.

The consumed name B and the newly accounted environment outputs are different ledgers. For a superposition $\sum_z \alpha_z|z\rangle$, the complete output $\sum_z \alpha_z|z\rangle_E|z\rangle_F$ still contains its phases jointly. Neither classical-looking marginal need contain them.

Corollary. A genuinely classical-output channel on a nontrivial full quantum code cannot have a complete pure dilation with one-dimensional environment.

Proof. With a one-dimensional environment, its complete dilation would be an isometry into the returned output alone. Theorem would supply an exact decoder. This contradicts the classical-output obstruction proved in Theorem below.

Thus deleting every *program name* for a residual does not prove that every unobserved physical degree of freedom has disappeared.

Keeping the residual record changes the access task

If the residual record is returned instead of dropped, the observer receives a larger interface. Its recovery possibilities can improve.

Example. For a qubit, let

$$V|\psi\rangle = \frac{1}{\sqrt{2}}(|\psi\rangle_R|0\rangle_B + Z|\psi\rangle_R|1\rangle_B),$$

where $Z = diag\ (1, -1)$. Before residual readout,

$$\mathcal{N}(X) = \frac{1}{2}(X + ZXZ).$$

Reading B gives outcome x with probability $1/2$ and branch map $X \mapsto Z\^{}xXZ\^{}x/2$. A decoder with R and the record applies $Z\^{}x$ and recovers exactly. Dropping the record leaves the original dephasing channel.

Proof. The two residual basis states are orthogonal, so $V\^{}\{\dagger\}V = I$. Taking their partial trace gives the displayed channel. The measurement bra $\langle x|_B$ extracts $Z\^{}x|\psi\rangle/\sqrt{(2)}$. Since $(Z\^{}x)\^{}2 = I$, correction followed by summing the two branches returns X. Every calculation is an operator identity and retains references.

This example does not contradict the record-only obstruction: the assisted decoder also receives the quantum register *R*.

The terminal readout is an added instruction with an added access permission. It is not an endpoint implied by autonomous dynamics. Partial trace is operational forgetting, not proved physical annihilation. Termination does not prove an explosion, an energy release, permanent absence of remnants, or the absence of future dynamics. Earlier records left untouched by the terminal operation are not retroactively erased.

Row 6: thermality, classical records, and information loss

The record-only obstruction

Theorem. Let a channel from *C* retain only a finite classical record:

$$\mathscr{F}(X) = \sum_x \mathrm{Tr}\,(F_x X)\mathrm{kb}\,xx_K, \qquad F_x \geq 0, \quad \sum_x F_x = I_C.$$

For every CPTP decoder $\mathscr{D}: \mathscr{L}(K) \to \mathscr{L}(C)$,

$$d_\diamond(\mathscr{D}\mathscr{F}, \mathrm{id}_C) \geq 1 - \frac{1}{d}.$$

The same bound holds for every measure-and-prepare channel. It is sharp: computational readout followed by computational-basis preparation attains it.

Proof. Write $\sigma_x = \mathscr{D}(kb\ xx)$. Then

$$\mathscr{D}\mathscr{F}(X) = \sum_x \mathrm{Tr}\,(F_x X)\sigma_x.$$

Its normalized Choi state is

$$\omega_{QC} = \frac{1}{d}\sum_x F_x^{\mathrm{T}} \otimes \sigma_x,$$

which is separable across $Q \mid C$. For normalized product vectors,

$$|\langle \Phi_d \mid a \otimes b\rangle|^2 \leq \frac{1}{d}$$

by Cauchy–Schwarz. Convexity gives $\langle \Phi_d|\omega_QC|\Phi_d\rangle \leq 1/d$. Testing the Bell projection therefore gives

$$d_{\mathrm{tr}}(\omega_{QC}, |\Phi_d\rangle\langle\Phi_d|) \geq 1 - \frac{1}{d}.$$

This is one admissible referenced channel test, proving . The argument already covers a general measure-and-prepare output.

For sharpness, let

$$Z_d = \sum_{j=0}^{d-1} e^{2\pi i j/d}\mathrm{kb}\,jj.$$

Computational dephasing satisfies

$$\Delta_d = \frac{1}{d}\sum_{k=0}^{d-1} \mathrm{Ad}_{Z_d^k}.$$

Hence

$$\|\mathrm{id}_C - \Delta_d\|_\diamond \leq \frac{1}{d}\sum_{k=1}^{d-1} \|\mathrm{id}_C - \mathrm{Ad}_{Z_d^k}\|_\diamond \leq 2\left(1 - \frac{1}{d}\right).$$

The lower bound already proved gives equality after dividing by two.

Measure-and-prepare channels are entanglement breaking [HorodeckiShorRuskai]. The quantitative comparison is also stated in [CheungHawk04B]. The proof does not prohibit measurement-assisted recovery with a surviving quantum output, as Example shows.

A finite adaptive program whose *only returned output* is classical has form . Indeed, the adjoint image of each classical output projector is an effect *F_x*, and trace preservation makes their sum the identity. Adaptivity and

unlimited classical storage do not evade the theorem. An additional input-dependent quantum memory would change its premise.

A useful calibration: a constant accessible channel

Lemma. Set

$$q_d = 1 - \frac{1}{d^2}.$$

If $\mathcal{N}$ is a replacement channel, then

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}) = q_d.$$

Proof. Every decoder of a replacement channel is a replacement channel $\mathcal{T}_{\tau}$. Its Bell-input output is $I_{Q}/d \otimes \tau$, with Bell overlap d^{-2}. Thus every decoder has error at least $1 - d^{-2}$.

For the matching upper bound, the decoder can prepare I_C/d. The completely depolarizing channel is the uniform average of the d^2 Weyl unitary channels. Separating the identity term and bounding the remaining channel differences by two gives

$$d_{\diamond}(\mathrm{id}_C, \mathcal{T}_{I_C/d}) \le 1 - d^{-2}.$$

The Bell lower bound makes this an equality.

Corollary. If $d_{\diamond}(\mathcal{N}, \mathcal{T}_{\omega}) \le \eta$, then

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}) \ge \max\{0, q_d - \eta\}.$$

Proof. For every decoder, contraction under postprocessing gives $d_{\diamond}(\mathcal{D}\mathcal{N}, \mathcal{D}\mathcal{T}_{\omega}) \le \eta$. The latter decoded comparison channel is a replacement. Lemma and the triangle inequality give $d_{\diamond}(\mathcal{D}\mathcal{N}, \mathrm{id}_C) \ge d - \eta$. Take the infimum.

The factor in is the *accessible* channel. Near constancy of the *complement* has the opposite recovery direction; see Section .

Nonconstant residual does not locate the entire code

For

$$V_{\mathrm{copy}}|i\rangle = |i\rangle_R |i\rangle_B,$$

each marginal channel is computational dephasing. Both are nonconstant, and neither recovers the full unknown code by Theorem . The relative phases remain available jointly.

By contrast, V_{hidden} has a nonconstant residual channel which *does* recover the entire code. Thus the assertion

``the complement is nonconstant, so neither factor recovers"

is false. The correct inference is only that R does not exactly recover. Residual-only recovery additionally requires constancy of $\mathcal{N}$.

Selected thermal data do not determine recovery

Proposition. Fix a finite Gibbs state

$$\tau_T = \frac{e^{-\beta H_T}}{\mathrm{Tr}\,(e^{-\beta H_T})}$$

and a purification $|\Theta\rangle_{TT'}$. Let $A \simeq B_1 \simeq C$, and choose

$$R = T \otimes A, \qquad B = T' \otimes B_1.$$

After fixed factor reordering, define isometries by

$$V_{\mathrm{rec}}|\psi\rangle = |\Theta\rangle_{TT'} |\psi\rangle_A |0\rangle_{B_1},$$
$$V_{\mathrm{hide}}|\psi\rangle = |\Theta\rangle_{TT'} |0\rangle_A |\psi\rangle_{B_1}.$$

Every observable on T has the same expectation in the two models, for every logical input. Nevertheless,

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}_{\mathrm{rec}}) = 0, \qquad \varepsilon_{\mathrm{rec}}(\mathcal{N}_{\mathrm{hide}}) = 1 - d^{-2}.$$

Proof. The factor Θ and the blank vector are fixed and normalized, so both maps are isometries. Their accessible channels are

$$\mathcal{N}_{\mathrm{rec}}(X) = \tau_T \otimes X_A, \qquad \mathcal{N}_{\mathrm{hide}}(X) = \mathrm{Tr}\,(X)\tau_T \otimes \mathrm{kb}\,00_A.$$

The T-reductions coincide. Tracing T recovers the first input exactly. The second channel is a replacement, so Lemma applies.

The proposition addresses selected thermal observations, not equality of the entire accessible channels. Once a complete channel has been specified, its Stinespring complementary information is fixed up to the usual environmental isometric equivalence. The gap is that a selected state or occupation law does not specify that complete channel [Stinespring,KSW].

Proposition. For every pure logical input, the accessible entropy is zero for both isometries in . Their optimal accessible recovery errors are respectively *1 − d^{−2}* and *0*.

Proof. The accessible output of either map is pure for a pure input. The recovery assertions follow from Example and Lemma .

A comparable selected-entropy separation is documented in the Cheung note Oai265-A [CheungOai265A]. Its use here neither assumes nor verifies the external catalogue family discussed by that note.

Thermal occupations do not by themselves prove that information is destroyed, that the whole accessible channel is a replacement, or that the complement is constant. A classical record is not an unknown quantum state returned with its reference correlations. No gravitational Page curve is obtained from the record channel, from one entropy value, or from the examples above [Hawking1975,Page1993].

The six-row operational certificate

Theorem. Under Definition and the explicitly specified finite Born readout convention:

• Complete isometric evolution preserves the code on *RB*. Residual-only relocation holds exactly when the accessible channel is a replacement; it is an additional property.

• The access declaration fixes the decoder domain. Ignored local operations on the residual do not change the accessible channel.

• Destructive Enter returns a classical record and the explicitly retained quantum remainder. It does not return the consumed quantum input.

• Finite scheduled emissions and adaptive readout trees define reference-preserving CPTP channels. Their storage and termination properties do not supply a mass, energy, or geometric law.

• Added residual readout followed by classical discard is exactly partial trace. It leaves the pre-existing accessible recovery error unchanged.

• Exact full-code recovery from an accessible quantum factor is equivalent to constancy of its complete complement. A purely classical returned record has recovery error at least *1 − 1/d*, while selected thermal or entropy data do not determine quantum recovery. These are statements about the declared finite interfaces. They do not identify those interfaces with a gravitational process.

Proof. Item 1 is Theorem , Corollary , and Example . Item 2 is Lemma . Item 3 is Definition and Proposition . Item 4 is Proposition , Theorem , and the separate defining data of the three ledgers. Item 5 is Theorem . Item 6 is Theorems and , together with Propositions and .

The geometric and dynamical objects needed for a gravitational identification are not constructed by any of these definitions or proofs. The last sentence is a scope statement, not a universal nonexistence theorem for possible future correspondence maps.

begingroup 5pt 1.12 Operational analogies and their identification limits. The first column names a narrative motif, not a theorem assumption.

toprule Narrative motif Precise finite object What is not established

midrule endfirsthead toprule Narrative motif Precise finite object What is not established

midrule endhead bottomrule endfoot

Classical collapse A supplied finite preparation followed by a complete isometry. Full relocation into *B* requires a replacement accessible channel or a separately exhibited relocation map. An event horizon, singularity, area theorem, or geometric no-return surface inferred from energy.

Event horizon The declared access split *R* | *B*, with all unreturned systems included in *B*. A metric-defined null surface, a causal boundary of spacetime, or gravitational area monotonicity.

Hawking radiation A declared outgoing quantum factor; after destructive Enter, a classical record and any explicitly retained quantum factors. A derivation of the Hawking temperature or spectrum, or an identification of thermal observations with a complete channel.

Radiation, mass loss, and shrinkage A finite schedule of transfer isometries and readouts, with separate storage, energy, erasure, and leakage accounting. A black-hole Hamiltonian, continuous mass loss, backreaction, or an internally constructed evaporation law.

Final burst; no remnant An additionally authorized readout of a finite residual, followed by discard of its record. The named residual output is consumed. Physical annihilation, an empty complete physical complement, an energetic burst, or an endpoint proved by termination.

Thermality and information loss A record-only obstruction and the full-code constant-complement recovery criterion. Input dependence can remain in the complete output. A Page curve, destruction under the complete isometry, or residual-only recovery merely from nonconstancy of the residual.

endgroup

Quantitative certificates and the additional endpoint obligations

This section is conditional and is not needed for the exact six-row certificate. Its sole non-elementary recovery input is explicitly identified.

Complement proximity is an upper certificate

The information–disturbance theorem of Kretschmann, Schlingemann, and Werner, converted to the half-diamond normalization used here, gives

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}) \le \min\{1, \sqrt{2\delta_{\mathrm{full}}(\mathscr{E})}\}.$$

This is an external mathematical theorem, not a gravitational input [KSW]. It expresses the fact that a nearly input-independent complete environment permits a nearly reversible accessible channel. It must not be confused with the accessible-output lower bound . The same directional distinction is retained in En07-B and En07-C [CheungEn07B,CheungEn07C].

Lemma. For channels Φ, Ψ with input dimension d,

$$d_{\diamond}(\Phi, \Psi) \le \frac{d}{2}\|J_{\mathrm{n}}(\Phi) - J_{\mathrm{n}}(\Psi)\|_1.$$

Proof. Every pure input with a d-dimensional reference can be written

$$|\psi\rangle = (L \otimes I_C)|\Phi_d\rangle, \qquad \mathrm{Tr}\,(L^{\dagger}L) = d.$$

Its output difference is obtained from the normalized Choi difference by multiplying on the left by $L \otimes I$ and on the right by $L^{\dagger} \otimes I$. Its trace norm is therefore at most

$$\|L\|_{\infty}^2 \|J_{\mathrm{n}}(\Phi) - J_{\mathrm{n}}(\Psi)\|_1 \le d\|J_{\mathrm{n}}(\Phi) - J_{\mathrm{n}}(\Psi)\|_1.$$

Convexity handles mixed inputs. Taking the referenced supremum and dividing by two proves the claim.

Corollary. Suppose a finite approximation $\widehat{\mathscr{E}}$ has been placed in the same complete residual output space as $\mathscr{E}$ by a specified isometric embedding, and a proved allowance gives

$$d_{\diamond}(\mathscr{E}, \widehat{\mathscr{E}}) \le b.$$

For a rigorously specified residual state $\hat{\sigma}$, put

$$e = \frac{1}{2}\left\| J_{\mathrm{n}}(\widehat{\mathscr{E}}) - I_Q/d \otimes \widehat{\sigma} \right\|_1.$$

Then

$$\delta_{\mathrm{full}}(\mathscr{E}) \le \min\{1, b + de\},$$
$$\varepsilon_{\mathrm{rec}}(\mathscr{N}) \le \min\{1, \sqrt{2(b + de)}\}.$$

Proof. Lemma gives $d_{\diamond}(\mathscr{E}, \mathscr{T}_{\hat{\sigma}}) \le de$. Add b by the triangle inequality and take the replacement-state infimum. Equation proves the recovery bound.

The dimension factor multiplies the normalized Choi estimate, not the already diamond-normalized allowance b. A related dimension-explicit certificate is documented in Hawk04-A [CheungHawk04A]. Neither a formula for such a certificate nor a finite approximation proves that its numerical value is small.

Four same-model endpoint obligations

An endpoint claim requires more than the termination of a finite tree. The following explicit package follows the distinction in En07-C, with En07-B as its companion comparison [CheungEn07C] [CheungEn07B].

Definition. An endpoint package for the fixed code supplies:

• A justified complete pure preparation $P_0: C \to H_{tot}$.

• A specified input-independent unitary evolution U_t on that complete system, on a justified time domain.

• Output identifications $F_t: H_{tot} \to R_t \otimes B_t$, factorwise isometries

$$a_t : R_t \to R_*, \qquad b_t : B_t \to B_*,$$

and an endpoint isometry $V_{*}: C \to R_{*} \otimes B_{*}$ such that

$$V_t = (a_t \otimes b_t) F_t U_t P_0, \qquad \nu_t = \min\{1, \|V_t - V_*\|_\infty\} \to 0.$$

The comparison maps preserve the access roles.

• A state $\sigma_{*} \in \mathscr{D}(B_{*})$ and a proved bound

$$d_{\diamond}(\mathscr{E}_*, \mathscr{T}_{\sigma_*}) \le \zeta, \qquad \mathscr{E}_* = \mathrm{Tr}_{R_*} \mathrm{Ad}_{V_*}.$$

For physical application, these data must be justified within one compatible microscopic model.

Theorem. Under Definition , let $\mathscr{N}_t = Tr_{B_{*}} Ad_{V_t}$ and $\mathscr{N}_{*} = Tr_{B_{*}} Ad_{V_{*}}$. Then

$$d_{\diamond}(\mathscr{N}_t, \mathscr{N}_*) \le \nu_t, \qquad d_{\diamond}(\mathscr{E}_t, \mathscr{E}_*) \le \nu_t.$$

Under the external input ,

$$\varepsilon_{\mathrm{rec}}(\mathscr{N}_t) \le \min\{1, \nu_t + \sqrt{2\zeta}\}.$$

If $\zeta = 0$, there is a single exact endpoint decoder $\mathscr{D}_{*}$ satisfying

$$d_{\diamond}(\mathscr{D}_* \mathscr{N}_t, \mathrm{id}_C) \le \nu_t.$$

Proof. For common-output isometries V, W and a referenced input state, write the output difference as

$$(V - W)\rho V^\dagger + W\rho(V^\dagger - W^\dagger),$$

with reference identities understood. Its trace norm is at most $2\|V - W\|_{\infty}$. Partial trace contracts trace distance, proving .

For every fixed decoder, postprocessing contraction changes its recovery error by at most ν_t. Taking the decoder infimum and using at the endpoint proves . If $\zeta = 0$, Theorem supplies $\mathscr{D}_{*}$ and contraction gives .

The factorwise embeddings cannot move an inaccessible logical register into R_{*}. That would change the access task rather than certify it. No gravitational model satisfying Definition is constructed in this manuscript. The theorem is a sufficient conditional certificate, not a claim that its hypotheses have been discharged or a universal characterization of all possible evaporation descriptions.

A worked finite schedule and the quantum-computer narrative

A concrete transfer schedule

Proposition. Let *C* consist of *n* qubits. Prepare a blank *n*-qubit register *M* and swap the code into it, leaving the old visible register blank. For each *j*, swap *M_j* into a fresh blank outgoing qubit *S_j*. Before outgoing readout, the first *k* emitted qubits together with the untransferred part of *M* contain the entire input, up to fixed register permutations and blank factors.

If each *S_j* is destructively read immediately, the record after *k* emissions is

$$\mathscr{F}_k(\rho) = \sum_{x \in \{0,1\}^k} \mathrm{Tr}\,[(\mathrm{kb}\, xx \otimes I_{2^{n-k}})\rho]\,\mathrm{kb}\, xx.$$

An added destructive readout-and-drop of the remaining quantum core does not change .

Proof. Every SWAP is unitary and only relocates a tensor factor. After *k* transfers, the logical tensor factors have been permuted into *S_1, …, S_k, M_{k + 1}, …, M_n.* All transferred positions of *M* are blank. This proves complete preservation, including on entangled inputs.

The computational measurement effects on the transferred factors pull back to *kb xx* ⊗ *I* on the original code. This gives . The terminal operation acts only on the remaining quantum factors and returns none of its outputs, so Theorem leaves the earlier record unchanged.

With all *n* outgoing qubits retained coherently, the observer can recover the whole code. With all of them read and only their classical record returned, Theorem applies. The scheduled core can become blank, but the measurement apparatus in a complete dilation must still be accounted for.

The example is not an emission model selected by gravity. Its purpose is to demonstrate the exact instructions needed for the analogy. A separately prepared Gibbs ancilla could produce additional temperature-labelled records without changing this logical transfer argument.

Imagining the processes on a quantum processor

Imagine a processor on which a finite code has already been prepared. Calling that code a star does not assert that every mode of a physical field has been encoded. A specified unitary now transfers the logical state to an internal register. The outside observer is denied that register. The machine has not created a geometric horizon: it has executed a relocation and changed what the observer can address.

The internal controller next exposes an outgoing register through a specified transfer isometry. This is an explicit interface change, not unexplained leakage through a fixed inaccessible cut. The observer can retain that register coherently or apply Enter. In the latter case, the quantum name is consumed and a bit string is returned. The readout can condition the unobserved state, although its average cannot signal to an untouched register.

A finite sequence of these transfers can make the named source core smaller. The complete inaccessible accounting need not become smaller with it: uncollected outputs and the apparatus used for destructive readout still belong somewhere in a complete dilation. An external control program may attach a temperature-like label or an increasing readout rate. Those labels do not establish heat, mass loss, or a gravitational equation.

Finally, suppose the controller is given permission to read the remaining finite residual. It performs Enter and discards the resulting record. The residual name disappears from the returned program interface. That is the exact content of readout-and-drop. The operation does not improve the channel already delivered to the observer, and it cannot erase earlier records stored elsewhere.

The program counter then reaches the end. Under the imaginative overlay, the sequence resembles collapse, outgoing radiation, shrinkage, and a terminal burst. Mathematically, it is a schedule of isometries and instruments. If every output name is discarded, the final observable channel is constant. If earlier records remain, they may still carry classical input dependence. In a complete isometric description, the omitted degrees of freedom have not been proved annihilated.

Most importantly, the location of recoverable information must be established, not narrated. For the explicit hidden-code map, the residual alone initially recovers. For a general isometry, recovery may require the joint output. For a record-only interface, unknown quantum coherence and reference entanglement are not returned.

These are the precise operational facts that make the analogy useful without turning it into an identification.

Standing refusals and conclusion

The manuscript establishes finite channel statements with declared access and output types. Its conclusions retain exactly that force.

• **No gravitational construction.** No metric, causal horizon, singularity theorem, area law, gravitational Hamiltonian, backreaction equation, or Planck-scale endpoint is constructed.

• **No spectral derivation.** Hawking's spectrum and temperature remain external semiclassical inputs. A thermal record is not silently promoted to a replacement channel.

• **No physical annihilation claim.** Consuming a register name and tracing a factor are operational statements. A complete residual is not proved empty merely because a named core has been read and dropped.

• **No entropy-to-recovery identification.** A selected entropy, a finite entropy turnover, and a referenced full-code recovery certificate are different objects. No gravitational Page curve is constructed.

• **No automatic thermodynamic cost.** Channel equality does not specify a memory, bath, battery, heat flow, or reset cycle. Energy, erasure cost, and leakage remain separate ledgers.

• **No hidden admission or implementation claim.** The inherited source-status label O.1 remains NOT_CLOSED; this finite matrix argument does not update it. No inherited gravitational quantum coefficient is admitted. The mathematical use of quantum amplitudes here is not admission of that separate characteristic coefficient. No physical field-to-qubit receiver, fixed-gate compilation, proof-assistant verification, or numerical execution is asserted.

• **No documentary transfer.** Companion manuscripts and external catalogue families retain their own versions, models, and hypotheses. Their citation supplies neither a missing correspondence nor verification of an external family.

The corrected operational conclusion is therefore

$$\text{complete isometry} \Rightarrow \text{joint quantum restoration},$$
$$\text{constant complete complement} \Leftrightarrow \text{exact accessible full-code recovery},$$
$$\text{destructive Enter} \Rightarrow \text{a declared classical record},$$
$$\text{dropBit} \circ \text{Enter} = \text{partial trace on the surviving interface}.$$

Neither a complete isometry nor a terminal instruction list identifies the quantum computer with an evaporating black hole. The analogy is disciplined precisely when the additional physical construction remains visible as an additional obligation.

appendix

Correction and provenance ledger

Corrections incorporated into the statement

The following corrections are part of the mathematical formulation, not merely interpretive qualifications.

• An arbitrary complete isometry preserves the code on *RB*. It does not, by itself, place a recoverable whole code in *B*.

• A nonconstant residual obstructs exact recovery from *R*. It neither proves residual-only recovery nor proves that both factors fail separately.

• Accessible readout leaves the unconditioned residual marginal unchanged when its outcomes are ignored. Conditional residual states and joint correlations can change.

• A residual that never interacts with the accessible factor and returns no outputs cannot emit information across a fixed cut. A transfer or access change must be specified.

• Terminal Enter on a previously inaccessible residual requires an explicit new permission. It is not an *R*-only decoder instruction.

• Readout-and-drop cannot improve or further degrade the pre-existing untouched accessible channel: that channel is exactly unchanged.

• Empty program ownership is not empty physical environment. Measurement and discarded-record stores must be counted in a complete dilation.

• A record-only obstruction does not prohibit a decoder that also receives surviving quantum information.

• Thermal occupations are not a complete channel specification. Conversely, the standard equivalence of Stinespring complements is not being denied.

• Finite termination is a syntactic property, not a physical evaporation endpoint or a proof of permanent absence.

Version-pinned comparisons

The following identifiers document comparisons. They are not dependencies for the locally proved exact theorem.

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Hawk04-A@1 Complete restoration, exact complete-complement recovery, and the distinction between global preservation and outgoing recovery. The supplied excerpts do not identify its microscopic model with the register examples here.

Hawk04-B@2 Typed destructive readout, classical-output obstruction, readout-and-drop, and apparatus accounting. The exact statements used here have local proofs.

En07-A@1 Complete-access restoration, operational forgetting, and the separation of channel equality from thermodynamic erasure.

En07-B@1 The conditional endpoint direction and the distinction between accessible replacement and complementary decoupling. The source's open physical obligations are not discharged here.

En07-C@1 External typing of Hawking's spectrum and the four same-model endpoint obligations. Only the finite conditional comparison is used.

En04-A@1 Nonconstant complementary response does not locate an independently recoverable whole code or establish geometric backreaction.

Oai265-A@1 A documentary selected-entropy separation. It is a Cheung comparison note about an external catalogue, not verification of that catalogue's area-law claim.

En02-C@1 A separately specified streaming benchmark distinguishes information-theoretic recovery from executable recovery under a live-qubit budget. Its source-reported one-slot error is *1/2*, while a two-slot protocol is exact; these model-specific values are not premises here [CheungEn02C].

En02-D@4 Its prepared entropy sequence *(0, 1, 2, 2, 1, 0)* is a finite-model turnover, not an identified gravitational Page curve. No emission law or transmission parameter from that model is imported [CheungEn02D].

endgroup

Publication of the journal article [CheungEnter] does not assign publication dates or journal status to the separately versioned companion manuscripts. The Philosophy essay is an introductory source, not a registered gravitational theorem. No claim is made that the present source has been machine checked.

Why a fixed finite unitary proxy does not prove permanence

A useful additional separation concerns recurrence. The following statement is conditional on a fixed finite closed model; it does not apply automatically to open, measured, time-dependent, or infinite-reservoir dynamics.

Proposition. Let U be a unitary on a finite-dimensional space. There are positive integers $m_k \to \infty$ such that $\|U^{m_k} - I\|_{\infty} \to 0$. Consequently, for a fixed code preparation and fixed output factorization, the corresponding complete and reduced channels return arbitrarily close to their time-zero channels along a subsequence.

Proof. The unitary group in finite dimensions is compact. Thus some subsequence U^{n_k} converges. Pass to a

further subsequence with $n_{k+1} - n_k \geq k$, and set $m_k = n_{k+1} - n_k$. Then

$$U^{m_k} = U^{n_{k+1}}(U^{n_k})^\dagger \to I,$$

while $m_k \to \infty$. Multiplying by the fixed preparation gives convergence of the complete isometries in operator norm. The isometry-to-channel estimate in the proof of Theorem , followed by partial-trace contraction, gives the channel conclusion.

The proposition supplies no recurrence claim about an actual black hole. It shows why a fixed finite proxy, without additional limiting or access data, cannot be used to infer irreversible permanent release. A comparable fixed-proxy separation is documented in Hawk04-B [CheungHawk04B].

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