# Hawk04-B Black-Hole Information as an Access Problem: Enter, Quantum Recovery, and Complete Residuals

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## Abstract

We distinguish complete unitary preservation, subsystem quantum recovery, destructive observation, and operational discard in a reference-preserving channel formulation. A finite acyclic typed fragment represents allocation, specified unitary transformations, destructive computational readout, and classical forgetting. Quantum recovery returns a quantum register; terminal readout returns a classical record and is a different task.

For a complete isometry $V: C \to R \otimes B$, we give an explicit global recovery channel and prove that exact recovery from $R$ is equivalent to constancy of the complete complementary channel to $B$. A process retaining only classical outcomes cannot reconstruct an arbitrary $d$-dimensional quantum code with half-diamond error below $1 - d^{-1}$. Unobserved readout of $B$, followed by discarding its outcomes, is exactly partial trace over $B$ and leaves the radiation channel unchanged. Its coherent dilation retains the information omitted from the classical interface.

Independently, a finite-band quartic source–scalar-reservoir Hamiltonian is constructed and approximated by finite-dimensional proxies with explicit complete-channel allowances. Separate qubit dictionaries preserve the accessible–inaccessible partition. For a commuting four-Majorana control, the exact errors and complete residual leakage are

$$\varepsilon_{\mathrm{rec}}^{R}(t) = \frac{3 - \sqrt{1 - c_R(t)^2}}{4}, \qquad \varepsilon_{\mathrm{rec}}^{B}(t) = \frac{1 - c_R(t)}{2}, \qquad \delta_{\mathrm{full}}(t) = \max\left\{\frac{1}{2}, \frac{1 + 2c_R(t)}{4}\right\}.$$

Thus the complete output is reversible although neither output alone need recover the diary.

Access-preserving endpoint continuity and fixed-proxy recurrence are proved separately from syntactic termination. The gravitational implication remains conditional on a same-model unitary dilation, outgoing access construction, and constant endpoint residual. No positive noncommuting recovery window, physical field-to-memory receiver, quantum-gravity unitarity theorem, or gravitational evaporation endpoint is established.



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**The governing distinctions.**

complete preservation $\nRightarrow$ ?recovery from a chosen subsystem,
quantum recovery $\neq$ ?destructive classical observation,
operational discard $\neq$ ?proved physical annihilation,
finite-program termination $\neq$ ?a physical evaporation endpoint.

*Theresidualisanoutputledger, notadeletionmechanism.*

## Introduction

The black-hole information problem asks whether a proposed evaporation description is compatible with complete quantum evolution. Hawking's radiation calculation and information-loss proposal provide its historical setting [Hawking1975,Hawking1976]. A finite circuit semantics does not derive that physical evolution.

This paper supplies an operational formulation in which the following questions have distinct mathematical objects:

• Is the finite quantum code preserved by the complete evolution?

• Does the declared accessible subsystem admit a quantum decoder?

• What instrument produces a specified classical record?

• Which outputs are omitted from the observer's description?

• Does the same physical model have an access-preserving endpoint? Every code comparison retains arbitrary external references.

The exact correction and no-hiding principles are established results [KnillLaflamme,BraunsteinPati]. The approximate channel conversion used here is due to Kretschmann, Schlingemann, and Werner [KSW]. We prove the exact statements locally, identify the imported approximate theorem explicitly, and make no priority claim for these principles. Superoperator semantics for quantum languages also has established precedent [Selinger].

The contribution is the joint accounting of typed destructive readout, complete complementary outputs, access-preserving regulator certificates, and an exact dynamical negative control. In particular, a nonconstant residual is not promoted to a recoverable residual-only copy of the entire unknown code.

### Relation to the supplied Cheung manuscripts

Hawk04-A separates complete preservation from radiation-only recovery and supplies the predecessor restore map, exact correction structure, four-Majorana control, and endpoint distinctions [CheungHawk04A]. Those results are proved or refined below.

Landmark I characterizes Born probabilities and positive-branch destructive residual rays within its declared strategy class [CheungI]. En03-A specifies a minimal linear circuit interface and distinguishes that interface from assertions about a larger calculus [CheungEn03A]. Here the finite fragment and its channel semantics are defined independently. Neither a field-to-qubit preparation nor the full hypotheses of Landmark I's characterization are reconstructed from its excerpt.

Hawk02-E supplies the regulated quartic source–scalar architecture, finite-proxy comparison discipline, and separation of source energy from complete residual independence [CheungHawk02E]. Hawk02-D concerns a different quadratic fermionic Hamiltonian and its scattering endpoint [CheungHawk02D]. No quadratic survival law or scattering amplitude is spliced into the present scalar model.

The apparatus and process distinctions in Em03-C, En02-B, En02-C, and En02-E are maintained [CheungEm03C,CheungEn02B,CheungEn02C,CheungEn02E]. In particular, the existence of a decoder is distinct from its execution under a particular resource budget; consuming syndrome qubits is distinct from consuming the recovered quantum output.

The analytic and OpenAI catalogue comparisons are confined to Appendices and . They are documentary separations, not premises of the channel or Hamiltonian proofs.

### The successor-program correction

The phrase ``radiation recovery, then *Enter* on *R*'' requires a correction. Applying *Enter* directly to the radiation register produces a record, not an arbitrary reference-entangled diary. The correct recovery interface is

$$R\mathrm{rightarrow}\ \mathscr{D}C_{\mathrm{out}},$$

with quantum return. A prescribed observation may subsequently consume *C_out*. These two target channels are *id_C* and the chosen measurement channel, respectively.

Likewise, a linear program cannot silently leave a live residual wire out of its return statement. Section therefore

distinguishes:

• the complete program returning *RB*;

• the local decoder program $R \to C_out$;

• a joint program returning *C_outB*;

• the observer's radiation-only marginal;

• an explicitly added readout-and-drop implementation of that marginal in a finite qubit encoding. The last implementation is not inferred from the autonomous Hamiltonian.

## Complete channels, access, and distances

Let $C \simeq \mathbb{C}^d$, with $d \geq 2$, be the logical code. All output Hilbert spaces are separable. States are normal density operators, and channels are normal completely positive trace-preserving maps on trace-class operators. We abbreviate ``completely positive and trace preserving" as CPTP.

We use

$$d_{\mathrm{tr}}(\rho,\sigma) = \frac{1}{2}\|\rho-\sigma\|_1, \qquad d_\diamond(\Phi,\Psi) = \frac{1}{2}\|\Phi-\Psi\|_\diamond.$$

The diamond norm is the reference-stable trace-norm distance. For a finite input of dimension $d$, a reference of dimension $d$ suffices for channel discrimination; the usual finite-input conventions are as in [Watrous].

For an isometry

$$V : C \to R \otimes B,$$

define

$$\mathcal{V}(X) = VXV^\dagger, \qquad \mathcal{M}(X) = \mathrm{Tr}_B\, \mathcal{V}(X), \qquad \mathcal{C}(X) = \mathrm{Tr}_R\, \mathcal{V}(X).$$

The inaccessible output $B$ is *complete*: it includes uncollected modes, retained source degrees of freedom, displaced carriers, preparation purifiers, and inaccessible apparatus factors. A factor is not removed from this ledger merely because an observer does not control it.

For a state $\sigma$, the replacement channel is

$$\mathcal{T}_\sigma(X) = \mathrm{Tr}\,(X)\sigma.$$

Set

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}) = \inf_{\mathcal{D}\ \mathrm{CPTP}} d_\diamond(\mathcal{D}\mathcal{N}, \mathrm{id}_C),$$
$$\delta_{\mathrm{full}}(\mathcal{C}) = \inf_\sigma d_\diamond(\mathcal{C}, \mathcal{T}_\sigma).$$

The infimum in is over deterministic decoders. A heralded success branch, by itself, is not such a decoder.

**Definition.** Quantum recovery returns a register carrying the logical code, including its correlations with an arbitrary untouched reference.

A classical observation returns a state of a commutative algebra, possibly together with explicitly declared surviving quantum registers. Its target is the specified observation channel, not *id_C*.

For a fixed code basis, write $E_ij = |i\rangle\langle j|$. Every logical channel is determined by its action on all $E_ij$; agreement on populations $E_ii$ alone does not establish quantum preservation.

**Lemma.** Let $\Phi$, $\Psi$ have input dimension $d$, and define

$$|\Phi_d\rangle = \frac{1}{\sqrt{d}}\sum_{i=0}^{d-1} |i\rangle_Q |i\rangle_C, \qquad J_{\mathrm{n}}(\Phi) = (\mathrm{id}_Q \otimes \Phi)(|\Phi_d\rangle\langle\Phi_d|).$$

Then

$$d_\diamond(\Phi,\Psi) \leq \min\left\{1, \frac{d}{2}\|J_{\mathrm{n}}(\Phi) - J_{\mathrm{n}}(\Psi)\|_1\right\}.$$

For common-output isometries,

$$d_\diamond(\mathcal{V}_1, \mathcal{V}_2) \le \|V_1 - V_2\|.$$

The same allowance holds after a common partial trace.

CPTP pre- and postprocessing contract these distances. On fixed output spaces,

$$|\varepsilon_{\mathrm{rec}}(\mathcal{N}_1) - \varepsilon_{\mathrm{rec}}(\mathcal{N}_2)| \mathrm{led}_\diamond(\mathcal{N}_1, \mathcal{N}_2),$$

and

$$|\delta_{\mathrm{full}}(\mathcal{C}_1) - \delta_{\mathrm{full}}(\mathcal{C}_2)| \mathrm{led}_\diamond(\mathcal{C}_1, \mathcal{C}_2).$$

*Proof.* A normalized pure referenced input can, after an isometry on its reference support, be written as *(L ⊗ I)|Φ_d⟩*, where *Tr (L^{†}L) = d* and *‖L‖^2 ≤ d*. Multiplication of the normalized Choi difference by *L* and *L^{†}* bounds its trace norm by *d* times the Choi trace norm. Convexity handles mixed inputs. This proves .

For normalized vectors *u*, *v*,

$$d_{\mathrm{tr}}(|u\rangle\langle u|, |v\rangle\langle v|) = \sqrt{1 - |\langle u, v\rangle|^2} \le \|u - v\|.$$

Apply this to referenced isometric outputs and then use convexity and partial-trace contraction.

The contraction statements follow from the definition of the diamond norm. For each fixed decoder, the triangle inequality changes its error by at most the channel comparison allowance. Taking infima in both directions proves recovery stability. Distance to the replacement-channel set is likewise one-Lipschitz.

**Lemma.** Separate isometric embeddings of *R* and *B* preserve *ε_rec* and *δ_full*. They do not exchange accessible and inaccessible outputs.

*Proof.* If *J: H → H^{'}* is an isometry, its channel has the CPTP retraction

$$\mathcal{R}_J(Y) = J^\dagger Y J + \mathrm{Tr}\,((I - JJ^\dagger)Y)\tau_H,$$

where *τ_H* is any fixed state. Embedding candidate decoders or replacement states gives one inequality. Applying the retraction and contraction gives the reverse inequality. The two embeddings act on their respective local factors only.

**Lemma.** Suppose unitary channels *Ad_U_j* have orthogonal Bell outputs. For real coefficients *a_j*,

$$\frac{1}{2}\Big\|\sum_j a_j \mathrm{Ad}_{U_j}\Big\|_\diamond = \frac{1}{2}\sum_j |a_j|.$$

In particular,

$$\inf_{\mathrm{sigmad}} _\diamond(\mathrm{id}_C, \mathcal{T}_\sigma) = 1 - d^{-2} =: a_d.$$

Every channel from *C* has recovery error at most *a_d*.

*Proof.* A Bell input gives a diagonal output difference with eigenvalues *a_j*, proving the lower bound. Each unitary channel has diamond norm one, so the triangle inequality gives the upper bound.

For any replacement state *σ*, the Bell output is *I_Q/d ⊗ σ*. Its overlap with the target Bell projector is *d^{−2}*. That projector therefore gives the lower bound *1 − d^{−2}*.

The *d^2* Weyl unitaries *X^pZ^q* have orthogonal Bell outputs, and their uniform twirl is *𝒯_{I_C/d}*. The preceding equality gives the matching upper bound. For an arbitrary channel, the decoder which ignores its input and prepares *I_C/d* attains this same allowance.

## Complete preservation, exact correction, and no-hiding

**Theorem.** Let *J: C → H_in* be an isometry, let *|Ω⟩* be a fixed normalized ancillary state, and let *U* be unitary on the complete physical input space. Then

$$V|\psi\rangle = U(J|\psi\rangle \otimes |\Omega\rangle)$$

is an isometry. Its complete channel has the CPTP left inverse

$$\mathcal{L}(Y) = V^\dagger Y V + \mathrm{Tr}\,((I - VV^\dagger)Y)\tau_C, \qquad \tau_C = I_C/d.$$

For every trace-class $X$, including operators on a code–reference space,

$$\|(V \otimes I)X(V^{\dagger} \otimes I)\|_1 = \|X\|_1.$$

For the Bell input, the complete output satisfies

$$I(Q : RB) = 2\log_2 d.$$

*Proof.* The composition gives *V^{†}V = I_C*. The two terms in are completely positive. Their traces sum to *Tr Y*, and *𝒱(VXV^{†}) = X*.

An isometric embedding preserves nonzero singular values, proving the trace-norm identity. For the Bell input, the complete output is pure with *d* equal Schmidt coefficients. Its reduced entropies are both *log_2 d*, and the joint entropy is zero.

If preparation is mixed, a fixed purification is included in the complete ancillary state. If preparation depends on the diary, the code-to-prepared-state map must itself be specified; it is not covered by a fixed-ancilla assumption.

The restore map is a complete-access inverse. Its existence does not imply an efficient inverse or a radiation-only inverse. It is the same global structure used in [CheungHawk04A].

**Theorem.** For an isometry $V: C \to R \otimes B$, the following are equivalent:

• one CPTP decoder on $R$ exactly recovers the entire code;

• $\mathscr{C} = \mathscr{T}_{\sigma}$ for some state $\sigma$;

• $\mathscr{C}(E_{ij}) = \delta_{ij}\sigma$;

•

$$J_{\mathrm{n}}(\mathscr{C}) = \frac{I_Q}{d} \otimes \sigma.$$

When these conditions hold, there are a separable Hilbert space $K$, a state $\sigma_K$, and an isometry $A: C \otimes K \to R$ such that

$$\mathcal{N}(X) = A(X \otimes \sigma_K)A^{\dagger}.$$

An exact decoder, defined on all trace-class operators on $R$, is

$$\mathscr{D}(Y) = \mathrm{Tr}_K\,(A^{\dagger}YA) + \mathrm{Tr}\,((I - AA^{\dagger})Y)\tau_C.$$

*Proof.* Linearity proves (ii) equivalent to (iii). Expansion of the Bell projector proves (iii) equivalent to (iv).

Assume (ii), and write the nonzero spectral decomposition

$$\sigma = \sum_{\alpha} p_{\alpha}|b_{\alpha}\rangle\langle b_{\alpha}|, \qquad p_{\alpha} > 0, \qquad \sum_{\alpha} p_{\alpha} = 1.$$

The index set is finite or countable. Constancy of the residual implies that every code vector is supported in $R \otimes$ *supp* $\sigma$. Write

$$V|i\rangle = \sum_{\alpha} \sqrt{p_{\alpha}}\,|r_{i\alpha}\rangle_R|b_{\alpha}\rangle_B.$$

The matrix-unit condition gives

$$\langle r_{j\beta}, r_{i\alpha}\rangle = \delta_{ij}\delta_{\alpha\beta}.$$

Hence

$$A(|i\rangle|\alpha\rangle) = |r_{i\alpha}\rangle$$

extends to an isometry, and *σ_K = ∑_{α} p_{α}|α⟩⟨α|* gives . Both terms of are CP; their traces sum to *Tr Y*. The decoder recovers every *E_ij*.

Conversely, let $L: R \to C \otimes G$ be a Stinespring isometry for an exact decoder. The normal Stinespring structure is the standard external dilation theorem [KSW,Watrous]. Set *Z = (L ⊗ I_B)V*. For basis input $|i\rangle$, the decoded marginal is the pure state $|i\rangle\langle i|$, so

$$Z|i\rangle = |i\rangle \otimes |\eta_i\rangle.$$

Exact recovery of $E_{ij}$ requires $\langle \eta_j, \eta_i \rangle = 1$. All $\eta_i$ are therefore one common normalized vector. Tracing the decoded register and $G$ yields $\mathscr{C}(E_{ij}) = \delta_{ij}\sigma$.

This is the exact correction/no-hiding structure associated with [KnillLaflamme,BraunsteinPati] and the normal form in [CheungHawk04A]. The local proof specifies the complete residual and the decoder's off-support completion.

**Corollary.** If $\mathscr{N}$ is constant on the entire code, $B$ exactly recovers that code. If $\mathscr{C}$ is constant, $R$ exactly recovers it. For $d \geq 2$, both complementary outputs cannot independently recover the entire unknown quantum code.

*Proof.* Interchange $R$ and $B$ in Theorem . An exactly recoverable output has constant complement, which cannot recover two distinct inputs.

**Remark.** A nonconstant $\mathscr{C}$ means that some logical inputs are distinguished by the residual channel. It does not mean that $B$ contains an independently recoverable copy of the entire code. Recovery may require joint access to $RB$. The exact control in Section demonstrates this.

A fixed pure residual gives $V|\psi\rangle = A|\psi\rangle|\omega\rangle_B$. A fixed mixed residual gives the syndrome form ; radiation may be mixed for a pure diary and still be exactly recoverable.

## A finite typed circuit semantics

### Types, ownership, and observable interfaces

Define the fragment

$$T ::= \mathrm{I} \mid \mathrm{Q} \mid \mathrm{Bit} \mid T \otimes T$$

with observable algebras

$$\mathscr{A}_{\mathrm{I}} = \mathbb{C}, \qquad \mathscr{A}_{\mathrm{Q}} = M_2(\mathbb{C}), \qquad \mathscr{A}_{\mathrm{Bit}} = \mathbb{C} \oplus \mathbb{C}.$$

Hybrid states are block diagonal on their classical registers. A bit is not an unrestricted qubit under a different name.

A judgement $P: T \to T^{\prime}$ consumes its input interface and returns its output interface. Sequential and tensor composition require matching interfaces. A live quantum wire is never duplicated. Every live quantum output is either returned or explicitly consumed by an admitted operation. Classical control acts blockwise.

The primitive channels are

$$\begin{aligned}
\mathrm{new} &: \mathrm{I} \to \mathrm{Q}, & \mathrm{new}(1) &= |0\rangle\langle 0|, \\
U &: \mathrm{Q}^{\otimes k} \to \mathrm{Q}^{\otimes k}, & \rho &\longmapsto U\rho U^{\dagger}, \\
\mathrm{Enter} &: \mathrm{Q} \to \mathrm{Bit}, & \rho &\longmapsto \sum_{x=0}^{1} \langle x|\rho|x\rangle |x\rangle\langle x|, \\
\mathrm{dropBit} &: \mathrm{Bit} \to \mathrm{I}, & (p_0, p_1) &\longmapsto p_0 + p_1.
\end{aligned}$$

Return commands designate the retained interface; they are not additional dynamics.

Only specified unitary constants are admitted. A semantic constant is not a claim of exact compilation over a fixed finite gate set, efficient synthesis, or physical realization. This independently defined ownership discipline agrees with the minimal interface supplied in [CheungEn03A].

### Destructive partial readout

Let $S$ be $s$ measured qubits, and let $T$ be the surviving quantum register. Set

$$M_x = \langle x|_S \otimes I_T, \qquad x \in \{0,1\}^s.$$

The observable instrument is

$$\mathscr{E}_S(\rho) = \sum_x |x\rangle\langle x|_K \otimes M_x \rho M_x^{\dagger}.$$

The measured names in $S$ are consumed. The surviving quantum output is $T$, not a renamed copy of $S$.

For a pure input,

$$p_x = \|M_x\psi\|^2, \qquad \psi_x = \frac{M_x\psi}{\sqrt{p_x}} \quad \text{if} p_x > 0.$$

No normalized residual is assigned to a zero-probability branch. Unnormalized branches suffice for complete-channel calculations.

These formulas are the destructive Born residuals in the supplied Landmark I characterization [CheungI]. The instrument is defined here directly. This does not reproduce that manuscript's uniqueness theorem without its full strategy-class hypotheses.

**Theorem.** For every finite acyclic instruction tree, and each fixed classical input block:

• the denotation is a CPTP channel to the declared hybrid output;

• there are quantum path maps *K_h* satisfying

$$\sum_h K_h^\dagger K_h = I;$$

• each reported output block is the sum of its unnormalized maps $X \mapsto K_hXK_h\^{\dagger}$;

• the same denotation holds with an arbitrary untouched quantum reference;

• every evaluation branch terminates within the finite height of the tree.

*Proof.* Allocation is an isometry, and a unitary gate is trace preserving. Destructive readout satisfies $\sum_x M_x\^{\dagger}M_x = I$. Classical control selects a complete channel on each classical block. Classical discard sums blocks without conditional normalization.

Induction composes path maps. Backward induction, using completeness at each readout node, gives $\sum_h K_h\^{\dagger}K_h = I$. Grouping histories with the same reported output gives the corresponding hybrid block. Tensoring each quantum path map with the reference identity proves complete positivity and the reference statement.

A finite acyclic tree has a finite maximum branch length.

The theorem is about the declared finite fragment. It is neither a complexity theorem nor a normalization theorem for an unprovided larger calculus. It uses the complete branch accounting emphasized in [CheungEn02C].

**Proposition.** A finite-dimensional isometry has a qubit encoding and a unitary extension after pure allocation.

A finite-dimensional CPTP channel can be represented by pure allocation, a specified unitary, destructive readout of its unused work register, discard of those work outcomes, and return of its quantum output.

*Proof.* Choose orthonormal basis dictionaries into sufficiently large qubit registers. Complete the columns of the encoded isometry to an orthonormal basis, obtaining a unitary extension.

For Kraus operators *D_j*, the map

$$L|\psi\rangle = \sum_j D_j|\psi\rangle|j\rangle_G$$

is an isometry because $\sum_j D_j\^{\dagger}D_j = I$. A unitary extension implements this coherent map. Readout of *G* and discard of the outcome give

$$X \text{longmapsto} \sum_j D_j X D_j^\dagger.$$

The designated logical output remains quantum.

When a logical dimension is not a power of two, the logical space is an encoded subspace of its qubit register. Every physical channel is completed on invalid padded inputs, for example by fixed-state preparation. Logical recovery is tested on *C*, including references; it is not an assertion about arbitrary invalid padded inputs.

An infinite-dimensional decoder or syndrome space need not admit a finite program. Theorem still supplies a normal CPTP decoder under its hypotheses.

## Explicit programs and their output ledgers

For a finite proxy, let *Q_C*, *Q_R*, *Q_B* denote qubit dictionaries for the logical code and the two outputs. These are

register abbreviations, with encoded-subspace conditions understood. Let *U_{body, t}* be a specified padded unitary whose restriction implements the prepared proxy isometry.

**Complete coherent preservation**
The complete program is

$$P_{\mathrm{coh},t} : Q_C \to Q_R \otimes Q_B.$$

Its body is

a := new;?(C,a) := EncodeAndPrepare(C,a);?(R,B) := Evolve_t(C,a);?returnQ(R,B);

Here new abbreviates the required finite pure allocation. The preparation and evolution are specified unitary extensions. Together they realize *U_{body, t}* on the encoded input.

There is no *Enter* in this coherent body. Its logical denotation is $\mathscr{V}_t$. The global restore map is ; in finite dimension it is realizable by Proposition . This is the typed reading of complete preservation in [CheungHawk04A,CheungEn03A].

The program begins after the carrier dictionary and preparation map have been defined. It constructs neither a quantum field nor a metric.

**A local quantum decoder**
For a specified finite CPTP decoder, the local program is

$$P_{\mathscr{D}} : Q_R \to Q_C^{\mathrm{out}},$$

with body

w := new;?(Cout,G) := DecoderUnitary(R,w);?y := Enter(G);?dropBit(y);?returnQ(Cout);

All unused work wires are included in *G*. The program consumes syndrome/work qubits, not the diary output. This interface is consistent with the quantum-return decoder programs in [CheungEn02B].

Applying this program locally to the coherent output gives

$$P_{\mathrm{keep},t} = (P_{\mathscr{D}} \otimes \mathrm{id}_{Q_B}) P_{\mathrm{coh},t} : Q_C \to Q_C^{\mathrm{out}} \otimes Q_B.$$

Its body returns (Cout,B). The observer who controls only the first returned factor sees

$$\widehat{\mathscr{D}\mathscr{N}_t}.$$

Thus radiation-only access is a declared marginal of a complete output, not an implicit rule deleting a live quantum wire.

**A strict one-register observable program**
If one additionally wants a finite program whose *observable* output is only *Q_C^out*, the residual must be consumed explicitly:

(R,B) := CoherentBody_t(C);?x := Enter(B);?dropBit(x);?Cout := Decode(R);?returnQ(Cout);

Its channel is again $\mathscr{D}\mathscr{N}_t$, by Theorem below.

This readout-and-drop is an external interface implementation. It is not a claim that the autonomous Hamiltonian measures *B*, nor that the omitted physical apparatus has disappeared. The complete dilation retains its apparatus outputs.

**Direct radiation observation and observation after recovery**
Directly measuring *R* denotes a channel

$$\mathscr{M}_R \mathscr{N}_t : C \to K,$$

not a quantum decoder.

For observation *after* recovery, let *F* be a specified basis-selection unitary on the encoded output and append

Cout := F(Cout);?x := Enter(Cout);?returnBit(x);

The observable channel is $\mathscr{M}_F \mathscr{D}\mathscr{N}_t$. Its intended target is $\mathscr{M}_F$, not *id_C*. For non-power-of-two codes, $\mathscr{M}_F$

includes the declared padded implementation and logical restriction.

**Proposition.** If

$$d_{\diamond}(\mathscr{D}\mathscr{N}_t, \mathrm{id}_C) \le \varepsilon,$$

then every prescribed terminal observation satisfies

$$d_{\diamond}(\mathscr{M}_F\mathscr{D}\mathscr{N}_t, \mathscr{M}_F) \le \varepsilon.$$

The converse for one chosen observation is false.

*Proof.* The upper bound is CPTP postprocessing contraction. Computational dephasing preserves the computational-basis observation exactly but annihilates off-diagonal matrix units. That observation equality does not imply quantum recovery.

Evaluating pre-written coherent code and executing the particular command *Enter* are therefore different operations. A coherent finite program may terminate without any destructive readout.

## Classical records cannot replace an unknown quantum diary

A measure-and-prepare channel has the form

$$\mathscr{E}(X) = \sum_x \mathrm{Tr}\,(F_x X)\sigma_x,$$

where *{F_x}* is a POVM and the *σ_x* are states. Such channels are entanglement breaking [HorodeckiShorRuskai]. The following proof gives the quantitative obstruction needed here.

**Theorem.** Let $\mathscr{E}$ be a measure-and-prepare channel from *C* to an arbitrary output, and let $\mathscr{D}$ be any CPTP decoder to *C*. Then

$$d_{\diamond}(\mathscr{D}\mathscr{E}, \mathrm{id}_C) \ge 1 - d^{-1}.$$

In particular, the bound applies to every finite adaptive process whose only retained output is classical.

Computational readout followed by basis-state preparation attains the bound.

*Proof.* The Bell-input output of $\mathscr{D}\mathscr{E}$ is separable across the reference–output partition. For a normalized product vector,

$$|\langle \Phi_d, u \otimes v\rangle|^2 \le \frac{1}{d}$$

by Cauchy–Schwarz. Convexity gives Bell overlap at most *1/d* for every separable state. Testing the Bell projector proves .

A channel to a finite classical register has the form

$$X \mathrm{longmapsto} \sum_x \mathrm{Tr}\,(F_x X)|x\rangle\langle x|.$$

A decoder from that register prepares a state for each outcome. Adaptive processing before the final classical output does not change this form.

For attainability, define

$$Z = \sum_{j=0}^{d-1} e^{2\pi \mathrm{i} j/d}|j\rangle\langle j|.$$

Computational dephasing is

$$\Delta_d = \frac{1}{d}\sum_{k=0}^{d-1} \mathrm{Ad}_{Z^k}.$$

Its *d* Bell outputs are orthogonal. Lemma gives *d_{◇}(Δ_d, id_C) = 1 − 1/d*.

For a qubit, every record-only reconstruction has error at least *1/2*. The hypothesis is that only the classical register is retained: a surviving quantum memory, or quantum side information supplied to the receiver, is a different access task.

A classical record may identify a basis label or support repeated-preparation tomography. It does not preserve an

arbitrary unknown quantum state and its reference correlations in one run.

## Residual readout, local forgetting, and the apparatus

### The explicitly added residual programs

For a finite encoded residual, define

$$P_{\text{readB}} : Q_R \otimes Q_B \to Q_R \otimes \text{Bit}^{\otimes b}$$

by

x := Enter(B);?return(R,x);

and define

$$P_{\text{forgetB}} : Q_R \otimes Q_B \to Q_R$$

by

x := Enter(B);?dropBit(x);?returnQ(R);

Neither program follows merely from the autonomous Hamiltonian.

The derived notation

$$\text{forgetQ} = \text{dropBit} \circ \text{Enter}$$

denotes this finite destructive-readout-and-forget implementation; it is not a primitive assertion of physical annihilation.

**Theorem.** For every joint state, including any external reference,

$$\sum_x (\langle x|_B \otimes I)\rho(|x\rangle_B \otimes I) = \text{Tr}_B \rho.$$

Consequently,

$$P_{\text{forgetB}} \mathcal{V}_t = \mathcal{N}_t.$$

Its radiation-only recovery error is exactly the pre-existing $\varepsilon_rec(\mathcal{N}_t)$.

More generally, every trace-preserving operation on $B$, with all its outputs ignored, leaves the state on the untouched reference and $R$ unchanged.

*Proof.* Partial trace in the computational basis is the sum of the diagonal $B$-blocks, which proves the first equality and .

For the general statement, let $\Lambda_B$ be the local channel. Its Heisenberg adjoint is unital. For every bounded observable $A$ on the untouched factors,

$$(\text{id} \otimes \Lambda_B^*)(A \otimes I) = A \otimes I.$$

All expectations on those factors are unchanged. This argument applies to normal channels on separable spaces; equivalently, in a Kraus representation, $\sum_j L_j\^{\dagger}L_j = I_B$.

This is no-signalling, not a statement that conditional radiation states cannot change. Conditioning on a supplied measurement outcome is a different operation.

For an infinite residual, partial trace remains well defined. The theorem does not assert a finite program that measures infinitely many modes.

**Proposition.** Suppose residual readout produces the hybrid channel

$$\mathcal{N}_{RK}(X) = \sum_x |x\rangle\langle x| \otimes \mathcal{N}_x(X), \qquad \sum_x \mathcal{N}_x = \mathcal{N}.$$

A deterministic decoder on its classical interface has the form

$$\mathcal{D}_{RK}(\mathcal{N}_{RK}(X)) = \sum_x \mathcal{D}_x(\mathcal{N}_x(X)),$$

where each $\mathcal{D}_x$ is CPTP on $R$. Moreover,

$$\varepsilon_{\text{rec}}(\mathcal{N}_{RK}) \le \varepsilon_{\text{rec}}(\mathcal{N}).$$

The inequality can be strict.

*Proof.* A channel on a direct sum of classical blocks acts by a CPTP channel on each block and adds the outputs. A decoder can ignore $K$, so every radiation-only strategy is available with $RK$ access. Example proves strictness.

### The coherent apparatus ledger

**Proposition.** For the destructive instrument , define

$$W_{\mathrm{read}}|\psi\rangle = \sum_x |x\rangle_K |x\rangle_E \otimes M_x|\psi\rangle.$$

This is an isometry. Tracing $E$ gives the observable hybrid readout instrument. The complete output $KET$ preserves the input globally, whereas the classical interface $K$, by itself, generally does not.

*Proof.* The identities $\langle x, y\rangle = \delta_{xy}$ and $\sum_x M_x^{\dagger} M_x = I$ give $W_{read}^{\dagger} W_{read} = I$. Tracing the orthogonal $E$-labels removes cross-outcome terms and yields . Complete recoverability follows from Theorem .

The physical carrier $K$ in the coherent dilation is a quantum system. It becomes the observable $Bit$ interface only after the inaccessible factor $E$ is omitted. A coherent superposition across outcome labels is not a state of the classical algebra.

For a fully measured qubit,

$$a|0\rangle + b|1\rangle \longmapsto a|0\rangle_K|0\rangle_E + b|1\rangle_K|1\rangle_E.$$

Neither marginal retains the unknown phase; joint $KE$ access does. There is no conflict with no-hiding, because neither marginal channel is constant on the entire qubit code.

Classical discard may be dilated by moving the record carrier into another inaccessible register. Under complete unitary implementation, that register and the other apparatus factors remain in the ledger. The observable trace does not prove physical disappearance.

This accounting is compatible with the joint-apparatus and coherent comparison discipline in [CheungEm03C,CheungEn02C]. No apparatus model from those manuscripts is silently substituted for the dilation specified here.

### Two exact destination tests

**Example.** For

$$V|\psi\rangle = |0\rangle_R|\psi\rangle_B,$$

radiation is constant and the complete residual exactly recovers the diary. Computational readout of $B$ retains only basis populations; its bits cannot replace the diary by Theorem . Readout-and-drop leaves radiation constant. A complete unitary measurement dilation remains globally reversible.

**Example.** For a qubit, consider

$$V|\psi\rangle = \frac{1}{\sqrt{2}}(|\psi\rangle_R|0\rangle_B + Z|\psi\rangle_R|1\rangle_B).$$

The radiation channel is $\Delta_Z = (id + Ad_Z)/2$, with optimal recovery error $1/2$. Computational readout of $B$ gives a uniform bit $x$. Supplying $x$ to the observer and applying $Z^x$ to $R$ gives exact quantum recovery. The bit alone contains no diary information.

*Proof.* The unnormalized branch is

$$\rho \longmapsto \frac{1}{2} Z^x \rho Z^x.$$

Correction by $Z^x$, followed by summation, returns $\rho$. Without correction, summation returns $\Delta_Z(\rho)$. Each outcome has probability $1/2$, independently of the input.

The two inputs $|+\rangle$ and $|-\rangle$ have identical dephased outputs. If a decoder has error $\varepsilon$, the triangle inequality and contraction imply $1 \le 2\varepsilon$. The identity decoder attains $1/2$ by Lemma .

This example separates classical side information supporting a quantum correction from a classical replacement for the quantum diary.

## An independently specified regulated microscopic model

The programming semantics does not construct the Hamiltonian below. The model follows the regulated quartic source–scalar architecture in [CheungHawk02E,CheungHawk04A]. We use units $\hbar = 1$.

### Frozen source and finite-band reservoir

Let $N = 2n \geq 4$, and let

$$\chi_i^\dagger = \chi_i, \qquad \{\chi_i, \chi_j\} = 2\delta_{ij} I$$

act on $\mathscr{H}_S = \mathscr{F}_f(\mathbb{C}^n)$. For frozen real coefficients $J_{ijkl}$, set

$$H_{\mathrm{bare}} = \sum_{i<j<k<l} J_{ijkl}\chi_i\chi_j\chi_k\chi_l, \qquad B_N = \sum_{i<j<k<l} |J_{ijkl}|,$$

$$H_S = H_{\mathrm{bare}} + B_N I, \qquad M_N = 2B_N.$$

Each quartic monomial is self-adjoint with norm one, hence

$$0 \leq H_S \leq M_N I.$$

Define

$$S_j = \mathrm{i}\chi_{2j-1}\chi_{2j}, \qquad O_{s'} = \frac{1}{n}\sum_{j=1}^{n} s_j' S_j, \qquad A = \alpha O_{s'} + \zeta O_{s'}^2,$$

where $s_j^{\{'\}} \in \{-1, 1\}$ and $\alpha, \zeta \in \mathbb{R}$. For the estimates, allow any fixed finite family of even self-adjoint source operators $A_a$, $1 \leq a \leq r$.

Fix

$$0 < \mu < \Lambda < \infty, \qquad \mathfrak{h} = L^2([\mu,\Lambda], \mathrm{d}\omega; \mathbb{C}^r), \qquad (h_B f)(\omega) = \omega f(\omega).$$

On $\mathscr{F}_s(\mathfrak{h})$, let

$$H_B = \mathrm{d}\Gamma(h_B), \qquad f_a(\omega) = \frac{\mathbf{e}_a}{\sqrt{2\omega}}, \qquad \Phi(f) = a(f) + a^\dagger(f).$$

Annihilation and the first slot of the one-particle inner product are conjugate-linear.

For frozen real $g_a$, set

$$H = H_S + H_B + V_{\mathrm{int}}, \qquad V_{\mathrm{int}} = \sum_{a=1}^{r} g_a A_a \otimes \Phi(f_a).$$

Identity factors are suppressed. Define

$$G = \sum_a |g_a| \|A_a\| \|f_a\|, \qquad G_\omega = \sum_a |g_a| \|A_a\| \left\|h_B^{-1/2} f_a\right\|.$$

The form-factor norms are

$$\|f_a\|^2 = \frac{1}{2}\log\frac{\Lambda}{\mu}, \qquad \left\|h_B^{-1/2} f_a\right\|^2 = \frac{1}{2}\left(\frac{1}{\mu} - \frac{1}{\Lambda}\right).$$

**Theorem.** The Hamiltonian is self-adjoint on $\mathscr{H}_S \otimes D(H_B)$, is bounded below, and satisfies the form inequality

$$H \geq \frac{1}{2}H_B - 2G_\omega^2 I.$$

It generates $U(t) = e^{-itH}$. Every isometric diary loading with a fixed pure reservoir preparation is globally preserved at each finite time.

*Proof.* On the finite-particle core,

$$\|a(f)\psi\| \leq \left\|h_B^{-1/2} f\right\| \left\|H_B^{1/2}\psi\right\|,$$

and the CCR imply

$$\left\|a^\dagger(f)\psi\right\|^2 = \|a(f)\psi\|^2 + \|f\|^2\|\psi\|^2.$$

Consequently,

$$\|V_{\mathrm{int}}\psi\| \le 2G_\omega \left\|H_B^{1/2}\psi\right\| + G\|\psi\|.$$

The square-root term has arbitrarily small relative bound with respect to *H_B*. Since *H_S* is bounded, the Kato–Rellich theorem gives self-adjointness on the stated domain [Kato]. These field estimates are also standard in spin–boson Hamiltonian constructions [HaslerHerbst].

For normalized form-domain vectors,

$$|\langle V_{\mathrm{int}}\rangle| \le 2G_\omega\sqrt{\langle H_B\rangle} \le \frac{1}{2}\langle H_B\rangle + 2G_\omega^2.$$

Together with *H_S ≥ 0*, this proves . The spectral theorem supplies the unitary group, and Theorem gives complete code preservation.

No ground-state, scattering, or evaporation theorem from [HaslerHerbst] is imported into this many-level model.

### Preparation, insertion, and access

Choose a normalized simultaneous spin eigenvector $|B_s\rangle$ of fixed source parity and let $\beta \ge 0$. Prepare

$$|\Omega_S\rangle = Z_s(\beta)^{-1/2} e^{-\beta H_S/2}|B_s\rangle, \qquad Z_s(\beta) = \langle B_s \mathrm{rverte}^{-\beta H_S}|B_s\rangle.$$

The bounds

$$e^{-\beta M_N} \le Z_s(\beta) \le 1$$

prove that the preparation is well defined. This is motivated by Euclidean-filtered SYK preparations [KourkoulouMaldacena], without importing a gravitational dictionary.

The microscopic diary is a qubit encoded in the even-parity sector of two carrier modes:

$$J_D|0\rangle = |00\rangle_D, \qquad J_D|1\rangle = |11\rangle_D.$$

With a fixed occupation ordering, exchange those modes with two declared source modes by a fixed second-quantized fermionic swap:

$$W|\psi\rangle = U_{\mathrm{sw}}(J_D|\psi\rangle \otimes |\Omega_S\rangle).$$

The map $W: C \to \mathscr{H}_S \otimes \mathscr{H}_D$ is an isometry. It uses one diary-independent unitary. The displaced source state and its correlations remain in the carrier; that carrier is retained in the residual.

Choose orthonormal packet functions $p_1, \ldots, p_q \in \mathfrak{h}$, and set

$$\mathfrak{r} = \mathrm{span}\{p_1, \ldots, p_q\}, \qquad R = \mathscr{F}_s(\mathfrak{r}),$$

$$B = \mathscr{H}_S \otimes \mathscr{H}_D \otimes \mathscr{F}_s(\mathfrak{r}^\perp).$$

With vacuum bath preparation and a fixed regrouping isometry $\mathcal{R}$,

$$V_t|\psi\rangle = \mathcal{R}\,(U(t) \otimes I_D)(W|\psi\rangle \otimes |\Omega_{\mathrm{bath}}\rangle).$$

All logical matrix units determine $\mathscr{N}_t$ and $\mathscr{C}_t$.

The source is prepared before the diary arrives. A heralded preparation may be conditioned on a diary-independent success event. A deterministic attempted protocol must retain failure outputs and compare its complete channel.

The channel theorems apply to arbitrary finite *d*. The specific two-mode diary construction above has *d = 2*.

## Finite proxies and access-preserving qubit dictionaries

### A complete finite-time regulator

Let *P_m* be the projection onto total bath occupation at most *m*. Put

$$Z_m(x) = \sum_{\ell=m+1}^{\infty} \frac{x^\ell}{\sqrt{\ell!}}.$$

**Theorem.** At fixed $\mu, \Lambda$, fixed packet family, and $T < \infty$, there is a finite-dimensional proxy with prepared isometry $V_t$, embedded in the same complete output space, such that uniformly for $0 \le t \le T$,

$$d_\diamond(\mathcal{V}_t, \widehat{\mathcal{V}}_t) \le b_{\mathrm{reg}} = \min\left\{1, 2Z_m(2GT) + \frac{Tm\Delta}{2}\right\}.$$

The same allowance holds for both reduced channels.

For $T > 0$ and $0 < b < 1$, sufficient choices are

$$m \ge 1, \qquad m + 1 \ge 4e(2GT)^2, \qquad m \ge 2 + \log_2(1/b), \qquad \Delta \le \frac{b}{Tm}.$$

These give $b_{reg} \le b$.

*Proof.* Use the interaction picture for $H_S + H_B$:

$$V_I(s) = \sum_a g_a A_a(s) \otimes \Phi(e^{\mathrm{i}sh_B} f_a).$$

On sectors of occupation at most $k$,

$$\|V_I(s)\psi\| \le 2G\sqrt{k+1}\,\|\psi\|.$$

An $\ell$-fold product starting from vacuum therefore has norm at most $(2G)^{\ell}\sqrt{(\ell!)}$. The ordered simplex has volume $t^{\ell}/\ell!$, so the $\ell$-th Dyson term is bounded by

$$\frac{(2Gt)^{\ell}}{\sqrt{\ell!}}.$$

These bounds are uniform over logical inputs and references.

Multiplication by any fixed power of the occupation number adds at most a polynomial in $\ell$, so the series converges in every fixed occupation-number graph norm on finite intervals. Since $h_B$ is bounded by $\Lambda$, this also controls the prepared energy domains. Termwise differentiation on these prepared vectors gives the Schrödinger equation; uniqueness for the self-adjoint Hamiltonian identifies the series with $U(t)$.

For the compressed Hamiltonian $P_m H P_m$, the Dyson terms agree with the exact terms through order $m$. Each additional projection is a contraction, and the same term bound applies to the compressed series. The prepared-isometry difference is therefore at most $2Z_m(2GT)$.

Partition $[\mu, \Lambda]$ into finitely many intervals $I_b$ of width at most $\Delta$, and replace $\omega$ on each interval by its midpoint, obtaining $\tilde{h}_B$. Then

$$\left\|h_B - \tilde{h}_B\right\| \le \Delta/2, \qquad \left\|P_m \mathrm{d}\Gamma(h_B - \tilde{h}_B) P_m\right\| \le m\Delta/2.$$

On the truncated space both Hamiltonians are bounded. Duhamel's formula adds at most $Tm\Delta/2$ to the prepared-isometry comparison.

The active one-particle space

$$\mathfrak{f} = \mathrm{span}\{1_{I_b} f_a, 1_{I_b} p_j : a, j, b\}$$

is finite dimensional, contains $\mathfrak{r}$ and all coupling vectors, and is invariant under $\tilde{h}_B$. Its orthogonal complement remains vacuum. Consequently the active proxy with total occupation cutoff $m$ is finite dimensional.

Its total-cutoff space embeds in

$$\mathscr{F}_{\mathrm{s}} \le m(\mathfrak{r}) \otimes \mathscr{F}_{\mathrm{s}} \le m(\mathfrak{f} \cap \mathfrak{r}^{\perp}).$$

This is an embedding, not an equality: the ambient tensor product also contains states with total occupation greater than $m$. Embedding the remaining inactive modes as vacuum gives the common complete output space. The source and carrier stay in $B$.

Lemma converts the prepared-isometry comparison to , including references and reduced channels.

Finally, let $x = 2GT$. The factorial bound gives

$$\frac{x^{m+1}}{\sqrt{(m+1)!}} \le \left(\frac{ex^2}{m+1}\right)^{(m+1)/2} \le 2^{-(m+1)}.$$

Later term ratios are at most $1/2$, whence $Z_m(x) \le 2^{-m}$. The occupation term $2Z_m(x)$ and the mesh term are

each at most *b/2* under the stated choices.

The theorem is a finite-time comparison at fixed positive infrared and finite ultraviolet cutoffs. Its constants do not provide uniform control under removal of either cutoff.

### Separate finite output dictionaries

Let $p = dim$ †. Finite ambient output dimensions are

$$D_R = \frac{q+m}{m}, \qquad D_B = 2^{n+2}\frac{p-q+m}{m}.$$

The factor *2^{n + 2}* accounts for the source and two-mode carrier. The total-cutoff proxy is a subspace of this ambient tensor product.

**Proposition.** There are separate basis isometries

$$E_R : \widehat{R} \to (\mathbb{C}^2)^{\otimes k_R}, \qquad E_B : \widehat{B} \to (\mathbb{C}^2)^{\otimes k_B},$$

where

$$k_R = \lceil \log_2 D_R \rceil, \qquad k_B = \lceil \log_2 D_B \rceil.$$

Their tensor product preserves the output partition, recovery error, and residual leakage. The encoded prepared proxy admits the coherent finite program of Section .

*Proof.* Choose separate orthonormal basis dictionaries. Lemma proves invariance of the two tasks.

The proxy Hamiltonian acts on its invariant finite total-cutoff subspace. Its unitary extends to the finite ambient tensor product and then to the padded qubit space. The preparation isometry has a unitary extension after allocation by Proposition .

This is an abstract dictionary and semantic realization. Relabelling packet modes as qubits does not construct a compact local detector or a coherent field-to-memory receiver. The separate receiver obligations in [CheungEm03C] are not discharged by this proposition.

### The energy ledger

**Proposition.** For the vacuum preparation above, define the source, bath, and interaction expectations by *E_S(t), E_B(t), E_I(t)*. On every finite interval,

$$E_S(t) + E_B(t) + E_I(t) = E_0,$$

where

$$E_0 = \mathrm{Tr}\,((H_S \otimes I_D) W \rho W^{\dagger}) \in [0, M_N].$$

Moreover,

$$0 \le E_B(t) \le 2M_N + 4G_\omega^2,$$

and

$$-\dot{E}_S = -\mathrm{i}\langle [V_{\mathrm{int}}, H_S] \rangle, \qquad \dot{E}_B = -\dot{E}_S - \dot{E}_I.$$

*Proof.* The initial bath energy and interaction expectation vanish. Autonomous evolution conserves total energy. Taking expectations in gives *E_B(t) ≤ 2E_0 + 4G_{ω}^2*.

The Dyson graph-norm estimates in Theorem give all fixed occupation moments on finite intervals. Since *h_B* is bounded and the source is finite, the field commutators in the displayed currents are controlled by those moments. Finite-particle approximation therefore justifies differentiation of their expectations. The Heisenberg equation gives the source current, and total energy conservation gives the second identity.

An externally imposed switching schedule would introduce a work term. No such schedule, measurement, or discard is hidden inside . Microscopic energy accounting is not a geometric evaporation law.

## Dimension-explicit recovery certificates

**The imported approximate conversion**

We use the finite-dimensional KSW information–disturbance theorem [KSW]. In Schrödinger-picture full-diamond normalization, its recovery direction gives, for any replacement state $\sigma$, a CPTP decoder satisfying

$$\|\mathscr{D}\mathscr{N} - \mathrm{id}_C\|_\diamond \le 2\sqrt{\|\mathscr{C} - \mathscr{T}_\sigma\|_\diamond}.$$

Taking infima and converting to the present convention gives

$$\varepsilon_{\mathrm{rec}}(\mathscr{N}) \le \sqrt{2\delta_{\mathrm{full}}(\mathscr{C})}.$$

Only finite complementary channels are used in this approximate step. The theorem supplies a channel conversion, not an estimate of interacting dynamics or an executable receiver.

**Theorem.** Suppose a finite complete proxy has comparison allowance

$$d_\diamond(\mathscr{V}, \widehat{\mathscr{V}}) \le b$$

on the common, access-preserving output space. For a specified finite residual state $\hat{\sigma}$, set

$$e = \frac{1}{2}\left\| J_{\mathrm{n}}(\widehat{\mathscr{C}}) - \frac{I_Q}{d} \otimes \hat{\sigma} \right\|_1.$$

Then

$$\delta_{\mathrm{full}}(\mathscr{C}) \le \min\{1, de + b\}.$$

There exists a finite proxy decoder whose full CPTP extension satisfies

$$d_\diamond(\mathscr{D}\mathscr{N}, \mathrm{id}_C) \le \min\{1, \sqrt{2de} + b\}.$$

If a specified finite CPTP decoder has

$$r_{\mathrm{dec}} = \frac{1}{2}\left\| J_{\mathrm{n}}(\widehat{\mathscr{D}\mathscr{N}}) - |\Phi_d\rangle\langle\Phi_d| \right\|_1,$$

then its full extension satisfies

$$d_\diamond(\mathscr{D}\mathscr{N}, \mathrm{id}_C) \le \min\{1, dr_{\mathrm{dec}} + b\}.$$

Any prescribed observation after quantum return has the same upper allowance relative to its observation target.

*Proof.* Partial trace of the complete comparison gives allowance $b$ for each reduced channel. Lemma bounds the proxy leakage by $de$. Embed $\hat{\sigma}$ into the complete residual space and use the triangle inequality to obtain .

Equation supplies a finite decoder with error at most $\sqrt{(2de)}$. Let $P_R$ project onto its finite accessible ambient subspace. Extend it to the full accessible space by

$$\mathscr{D}(Y) = \widehat{\mathscr{D}}(P_R Y P_R) + \mathrm{Tr}\,((I - P_R)Y)\tau_C.$$

The extension is CPTP and agrees on proxy outputs. Decoder contraction adds $b$, outside the square root, giving .

For the specified decoder, gives the proxy allowance $dr_{dec}$; the same extension adds $b$. Proposition proves the terminal-record statement.

Validated upper bounds on $e$ or $r_{dec}$ may replace their exact values. A finite decoder certificate must establish CPTP feasibility, not merely a favorable objective value. For a decoder with input dimension $D_R$, its normalized Choi conditions are

$$J_{\mathrm{n}}(\widehat{\mathscr{D}}) \mathrm{succeq}\, 0, \qquad \mathrm{Tr}_{C_{\mathrm{out}}} J_{\mathrm{n}}(\widehat{\mathscr{D}}) = \frac{I_R^{\wedge}}{D_R}.$$

The factor $d$ in the logical Choi conversion is not omitted. The regulator allowance $b$ is already a half-diamond allowance and is not multiplied by $d$.

**Proposition.** Let

$$L_m = M_N + m\Lambda + 2G\sqrt{m+1}.$$

Suppose a fixed finite decoder has a validated point certificate $r_{dec}(t_0) \le r_0$. Then its continuum extension

obeys, on the chosen finite interval,

$$d_{\diamond}(\mathscr{D}\mathscr{N}_t, \mathrm{id}_C) \le \min\{1, dr_0 + L_m|t - t_0| + b_{\mathrm{reg}}\}.$$

If *L_m > 0* and *dr_0 + b_reg < ε*, it has a nonempty certified interval

$$|t - t_0| < \frac{\varepsilon - dr_0 - b_{\mathrm{reg}}}{L_m},$$

intersected with the observation interval. If *L_m = 0*, the proxy channel is time independent.

*Proof.* The norm of the finite active Hamiltonian is at most *L_m*. Duhamel's formula and give proxy channel distance at most *L_m|t − t_0|*. Apply the fixed decoder after the point certificate and add the continuum comparison.

No positive noncommuting point certificate is supplied here. The existence of a finite input matrix does not itself validate an optimization result.

Compatible preparation, collection, and implementation allowances can be added by telescoping CPTP compositions, as in [CheungEn02B,CheungEn02C]. A scalar energy error, metric error, or entropy error is not added to a diamond budget without a conversion theorem. The flagged-cutoff identities in [CheungEn02E] retain their commuting-state hypotheses.

## The commuting four-Majorana control

### The general commuting obstruction

**Proposition.** Suppose

$$[H_S, A_a] = 0, \qquad [A_a, A_b] = 0$$

for every pair. For any isometric diary insertion and fixed bath preparation, the collected-reservoir channel is measure-and-prepare. Every deterministic radiation decoder has error at least *1 − d^{−1}*.

*Proof.* Choose simultaneous source eigenspace projectors *P_{λ}*. The evolution has blocks

$$U(t) = \sum_{\lambda} P_\lambda \otimes U_\lambda(t).$$

After tracing the source, cross-projector terms vanish. Including the retained carrier in the trace gives

$$\mathscr{N}_t(\rho) = \sum_{\lambda} \mathrm{Tr}\,(F_\lambda \rho)\sigma_{\lambda,t}, \qquad F_\lambda = W^{\dagger}(P_\lambda \otimes I_D)W.$$

The effects are positive and sum to *I_C*. Theorem applies.

If a channel is compared in diamond distance to such a control, the resulting statement is a lower bound:

$$\varepsilon_{\mathrm{rec}}(\mathscr{N}) \ge \max\{0, 1 - d^{-1} - d_{\diamond}(\mathscr{N}, \mathscr{N}_{\mathrm{QND}})\}.$$

Closeness to a commuting control is not a high-accuracy recovery certificate.

### An exact frozen realization

Fix

$$N = 4, \qquad J_{1234} = J > 0, \qquad A = \frac{S_1 + S_2}{2}, \qquad g \neq 0,$$

with one scalar reservoir species. For

$$c_j = \frac{\chi_{2j-1} + i\chi_{2j}}{2},$$

one has

$$S_j = 2c_j^{\dagger}c_j - I, \qquad H_S = J(I - S_1 S_2).$$

Prepare *|11⟩_S*, which is unchanged by Euclidean filtering. Swap the two-mode diary into the entire source. The retained carrier becomes the fixed state *|11⟩_D*. The logical source states are

$$s_0 = |00\rangle_S, \qquad s_1 = |11\rangle_S,$$

both with source energy zero, and with coupling eigenvalues *a_0 = −1*, *a_1 = 1*.

For *f(ω) = 1/√(2ω)*, define

$$\eta_i(t) = g a_i \frac{e^{-i\omega t} - 1}{\omega} f(\omega).$$

For the collected packet space $\mathfrak{r}$, put

$$\eta_{i,R} = P_{\mathfrak{r}} \eta_i, \qquad \eta_{i,U} = P_{\mathfrak{r}^\perp} \eta_i,$$

and

$$c_R(t) = \exp\left[-\frac{1}{2}\|\eta_{0,R} - \eta_{1,R}\|^2\right], \qquad z_R(t) = \sqrt{1 - c_R(t)^2}.$$

The collected coherent states have real positive overlap *c_R*.

**Theorem.** For this realization,

$$\mathcal{N}_t(\rho) = \rho_{00} |\eta_{0,R}\rangle\langle\eta_{0,R}| + \rho_{11} |\eta_{1,R}\rangle\langle\eta_{1,R}|,$$

$$\varepsilon^R_{\mathrm{rec}}(t) = \frac{3 - z_R(t)}{4} \geq \frac{1}{2},$$

$$\varepsilon^B_{\mathrm{rec}}(t) = \frac{1 - c_R(t)}{2},$$

$$\delta_{\mathrm{full}}(t) = \max\left\{\frac{1}{2}, \frac{1 + 2c_R(t)}{4}\right\}.$$

The complete output *RB* recovers the diary exactly.

If all reservoir modes are declared accessible, then

$$c_R(t) = \exp\left[-2g^2 \int_\mu^\Lambda \frac{1 - \cos(\omega t)}{\omega^3} \mathrm{d}\omega\right].$$

*Proof.* For source eigenvalue *a_i*, let *q_i = ga_ih_B^{−1}f*. The Weyl unitary $\mathcal{W}(q) = e^{a^{\dagger}(q) - a(q)}$ gives

$$H_B + g a_i \Phi(f) = \mathcal{W}(q_i)^\dagger H_B \mathcal{W}(q_i) - g^2 a_i^2 \|h_B^{-1/2} f\|^2 I.$$

The vacuum evolves to $|\eta_i(t)\rangle$, up to a phase. For *a_i = ±1*, the phases are common. Factoring the coherent state between collected and uncollected modes gives

$$V_t |i\rangle = |\eta_{i,R}\rangle_R \otimes |s_i\rangle_S |11\rangle_D |\eta_{i,U}\rangle.$$

Tracing the orthogonal source labels proves .

Write *σ_i = |η_{i, R}⟩⟨η_{i, R}|*. Their trace distance is *z_R*. For any effect $0 \leq F \leq I$,

$$\frac{1}{2}(\mathrm{Tr}\,(F\sigma_0) + \mathrm{Tr}\,((I - F)\sigma_1)) \leq \frac{1 + z_R}{2},$$

because the maximum of *Tr (F(σ_0 − σ_1))* is the trace of its positive part, namely *z_R*.

For any decoder, its Bell-input overlap with the ideal output is

$$F_e = \frac{1}{4}(\langle 0|\mathcal{D}(\sigma_0)|0\rangle + \langle 1|\mathcal{D}(\sigma_1)|1\rangle) \leq \frac{1 + z_R}{4}.$$

Indeed, the adjoint image of *kb 00* is an effect and its complement is the adjoint image of *kb 11*. The Bell-projector test therefore gives

$$d_\diamond(\mathcal{D}\mathcal{N}_t, \mathrm{id}) \geq \frac{3 - z_R}{4}.$$

Bosonic parity interchanges *σ_0* and *σ_1*. The optimal discrimination measurement can thus be chosen with symmetric errors, each *(1 − z_R)/2*, by symmetrizing an optimal measurement. Measuring and preparing the inferred logical basis state produces the channel

$$z_R \Delta_Z + (1 - z_R)\mathcal{T}_{I_2/2}.$$

Its Pauli probabilities are

$$p_I = p_Z = \frac{1 + z_R}{4}, \qquad p_X = p_Y = \frac{1 - z_R}{4}.$$

The four Pauli Bell outputs are orthogonal. Lemma gives error *1 − p_I = (3 − z_R)/4*, proving . The decoder is completed on the orthogonal complement of the two-state support.

For the residual, define

$$L_t|i\rangle = |s_i\rangle_S|11\rangle_D|\eta_{i,U}\rangle.$$

The source labels make *L_t* an isometry. Tracing *R* gives

$$\mathscr{C}_t = L_t \Delta_{c_R}(\,\cdot\,) L_t^\dagger, \qquad \Delta_c(\rho) = \begin{pmatrix} \rho_{00} & c\rho_{01} \\ c\rho_{10} & \rho_{11} \end{pmatrix}.$$

Retracting the range of *L_t* gives

$$\Delta_c = \frac{1 + c}{2}\mathrm{id} + \frac{1 - c}{2}\mathrm{Ad}_Z,$$

with error *(1 − c)/2*. For the lower bound, the residual outputs of $|+\rangle$ and $|-\rangle$ have trace distance *c*. If a decoder has error *ε*, contraction and the triangle inequality give *1 ≤ 2ε + c*. This proves .

By Lemma , residual leakage equals the leakage of *Δ_c*. This channel is Pauli covariant. Conjugating a replacement state by a Pauli leaves its distance to *Δ_c* unchanged after the corresponding input conjugation. Convexity therefore shows that twirling the replacement state to *I_2/2* cannot increase the distance; that replacement is optimal.

Relative to *𝒯_{I_2/2}*, the Pauli coefficient differences, in the order *I*, *Z*, *X*, *Y*, are

$$\frac{1 + 2c}{4}, \qquad \frac{1 - 2c}{4}, \qquad -\frac{1}{4}, \qquad -\frac{1}{4}.$$

Lemma gives

$$\delta_{\mathrm{full}} = \frac{3 + 2c + |1 - 2c|}{8} = \max\left\{\frac{1}{2}, \frac{1 + 2c}{4}\right\}.$$

Complete recovery follows from Theorem . Finally,

$$\|\eta_0 - \eta_1\|^2 = 4g^2 \int_\mu^\Lambda \frac{1 - \cos(\omega t)}{\omega^3}\mathrm{d}\omega,$$

which proves .

The radiation optimum is the negative control in [CheungHawk04A]. The residual formulas complete its destination accounting. For *0 < c_R < 1*, both local recovery errors are positive, although the complete output remains reversible. Neither marginal is constant on the whole code, so this is not a no-hiding counterexample.

**Proposition.** For this realization,

$$E_S(t) = 0, \qquad E_B(t) = g^2 \int_\mu^\Lambda \frac{1 - \cos(\omega t)}{\omega^2}\mathrm{d}\omega, \qquad E_I(t) = -E_B(t).$$

Every unobserved residual readout-and-drop leaves and unchanged.

*Proof.* The source stays in its zero-energy logical sector. The coherent bath energy is *‖h_B^{1/2}η_i(t)‖^2*, giving the integral. Initial total energy is zero, so conservation gives *E_I = −E_B*. The last statement is Theorem .

The bath energy is a dressing response of the initially undressed preparation. It is not a derived gravitational mass-loss law. The control is commuting, not a generic noncommuting or chaotic SYK recovery result. No scrambling or out-of-time-order estimate is used.

## The corrected destination ledger

The word ``bin'' denotes a declared residual subsystem. It does not denote a physical deletion mechanism. The table concerns unknown quantum codes, not only classical labels.

toprule Condition or interface Complete-channel conclusion Recovery task

midrule endfirsthead toprule Condition or interface Complete-channel conclusion Recovery task

midrule endhead bottomrule endfoot

Complete coherent body The code is preserved in *RB*. The program returns both quantum outputs; it contains no *Enter*. Global restore ; complete access is sufficient.

Constant complete complement Radiation has the syndrome form $A(X \otimes \sigma_K)A^{\dagger}$. The residual is logical-input independent. Local quantum decoder , followed by quantum return.

Constant radiation channel No-hiding places a recoverable copy of the whole code in the complete residual. Quantum recovery from *B*; classical readout of *B* is not generally that recovery.

Merely nonconstant complement Some logical information is distinguished by *B*. Residual-only recovery of the entire code does not follow. A local decoder requires a separate proof; joint recovery exists for the complete isometry.

Four-Majorana control Radiation is measure-and-prepare. For $0 < c_R < 1$, both individual recovery errors are positive, while *RB* is reversible. $\varepsilon_{rec}^{R} = (3 - \sqrt{1 - c_R^2})/4 \geq 1/2$; $\varepsilon_{rec}^{B} = (1 - c_R)/2$.

Residual readout with record retained Observable access is *RK*, with unnormalized conditional quantum branches. The complete apparatus also retains *E*. An assisted quantum decoder may use *RK*. Bits alone do not replace the diary.

Residual readout with record dropped The observable output is exactly the original channel $\mathcal{N}$, not a newly erased radiation channel. Exactly the pre-existing radiation-only recovery task.

All retained outputs converted to classical data The observable channel is measure-and-prepare after any classical-to-quantum reconstruction. Error at least $1 - 1/d$.

All observable outputs dropped The retained algebra is scalar. Any newly allocated fixed quantum output is a replacement channel. Optimal deterministic reconstruction error $a_d = 1 - d^{-2}$; the complete unitary dilation still has a global inverse.

Vanishing source energy alone It constrains a selected energy expectation, not the complete complementary channel. No radiation recovery certificate follows without a channel comparison.

Selected radiation entropy alone An entropy of selected inputs does not identify the destination of all logical matrix units and reference correlations. No decoder or residual-independence conclusion follows.

**Proposition.** A source can remain in one fixed zero-energy vacuum while an inaccessible memory retains the entire diary. Also, two complete isometries can give the same zero radiation entropy for every pure code input but have radiation recovery errors $0$ and $a_d$, respectively.

*Proof.* Let $B = S \otimes M$, choose $H_S|vac\rangle = 0$, and set

$$V|\psi\rangle = |0\rangle_R|\mathrm{vac}\rangle_S|\psi\rangle_M.$$

The source energy is identically zero, the complete residual recovers exactly, and radiation is constant. Its recovery error is $a_d$ by Lemma . The residual is not logical-input independent.

For the entropy comparison, take

$$V_{\mathrm{good}}|\psi\rangle = |\psi\rangle_R|0\rangle_B, \qquad V_{\mathrm{bad}}|\psi\rangle = |0\rangle_R|\psi\rangle_B.$$

For pure inputs, both radiation entropies are zero. Their recovery errors are $0$ and $a_d$.

These locally proved counterexamples implement the diagnostic refusals in [CheungHawk02E,CheungHawk04A]. They concern insufficient diagnostics, not a claim that every energy or entropy theorem is uninformative.

## Termination, endpoint continuity, and recurrence

### Finite evaluation is not an endpoint

For each fixed $t$, a finite proxy program may use the specified constant $U(t)$. The acyclic program terminates by Theorem . This says nothing by itself about $t \to \infty$, uniform regulator control, disappearance of a residual, or

existence of an outgoing receiver.

An outgoing scattering representation may define the target of a later finite receiver approximation. It need not be a terminal configuration of the finite syntax.

**Definition.** For time-dependent output partitions $R_t \otimes B_t$, choose local isometries

$$J_{R,t} : R_t \to R_\infty, \qquad J_{B,t} : B_t \to B_\infty.$$

Set

$$\overline{V}_t = (J_{R,t} \otimes J_{B,t}) V_t.$$

These identifications do not move an inaccessible diary-bearing factor into accessible radiation.

**Theorem.** Suppose, on a code basis,

$$\overline{V}_t |i\rangle \to V_\infty |i\rangle.$$

Then $V_{\infty}$ is an isometry and

$$a_t = \left\| \overline{V}_t - V_\infty \right\| \to 0.$$

Complete and reduced channels converge in half-diamond distance with allowance $a_t$. In particular,

$$|\delta_{\text{full}}(t) - \delta_{\text{full}}(\mathscr{C}_\infty)| \le a_t, \qquad |\varepsilon_{\text{rec}}(\mathscr{N}_t) - \varepsilon_{\text{rec}}(\mathscr{N}_\infty)| \le a_t.$$

If $\delta_{full(t)} \to 0$, the endpoint complete complement is constant and the endpoint radiation exactly recovers the code.

*Proof.* Finite-dimensionality of $C$ gives

$$\left\| \overline{V}_t - V_\infty \right\|^2 \le \sum_i \left\| (\overline{V}_t - V_\infty) |i\rangle \right\|^2 \to 0.$$

Inner products pass to the limit, so $V_{\infty}^{\dagger}V_{\infty} = I$. Lemma gives channel convergence and stability. Lemma identifies the embedded tasks with the original local tasks.

It remains to justify exact constancy when endpoint leakage is zero, even if the endpoint output is infinite dimensional. Choose states $\sigma_n$ with $d_{\diamond}(\mathscr{C}_{\infty}, \mathscr{T}_{\sigma_n}) \to 0$. Evaluation at $\tau_C$ gives

$$d_{\text{tr}}(\sigma_n, \mathscr{C}_\infty(\tau_C)) \to 0.$$

Replacement-channel distance equals the corresponding state distance. The triangle inequality therefore implies

$$\mathscr{C}_\infty = \mathscr{T}_{\mathscr{C}_\infty(\tau_C)}.$$

Apply Theorem .

If the constant endpoint residual is pure, the endpoint radiation channel is an isometric encoding. If it is mixed, the syndrome normal form applies. An infinite endpoint syndrome need not have a finite decoder program.

The access condition is essential. For $V\psi = |0\rangle_R \psi_B$, swapping the two outputs changes radiation recovery from impossible to exact. Such a swap is not an access-preserving identification.

The theorem refines the same finite-code endpoint structure in [CheungHawk04A]. The scattering endpoint of [CheungHawk02D] belongs to its quadratic model. In the supplied convention its outgoing one-particle map is $\Omega_{+}^{\dagger}J_S$, with the adjoint required by the domains of the wave operator. No interacting asymptotic-completeness theorem for follows from that comparison.

### Fixed finite proxies recur

**Proposition.** At fixed finite occupation cutoff and frequency mesh, the autonomous proxy returns arbitrarily closely to its initial prepared channel at arbitrarily late times. For initially vacuum accessible radiation,

$$\limsup_{t \to \infty} \varepsilon_{\text{rec}}(\widehat{\mathscr{N}_t}) = a_d = 1 - d^{-2}.$$

*Proof.* A finite-dimensional Hamiltonian has finitely many eigenvalues. Its phase flow lies in a compact torus. Simultaneous recurrence gives a sequence $t_j \to \infty$ for which the unitary approaches the identity. If the phase orbit

is periodic, use multiples of its period; otherwise compactness and the group property give arbitrarily late approximate returns.

Lemma yields prepared-channel recurrence. Initially the accessible bath is vacuum, so its logical channel is constant and has recovery error *a_d*. Recovery stability gives errors tending to *a_d* on the recurrence sequence. Every channel has error at most *a_d* by Lemma , proving the limsup identity.

This strengthens the fixed-proxy recurrence statement in [CheungHawk02E] and is consistent with [CheungHawk04A]. A finite-time approximation must precede a controlled outgoing limit; taking $t \to \infty$ at a fixed closed proxy is not a permanent-release theorem.

**The separate gravitational conditional**

**Corollary.** Suppose one physical gravitational model supplies:

• a justified quantum code and complete preparation/dilation;

• complete unitary evolution;

• a derived outgoing access and endpoint construction;

• proof that the endpoint complete residual is constant. Then outgoing radiation exactly recovers the code and its reference correlations.

If the entire nonradiative endpoint is one fixed pure state, the radiation channel is an isometric encoding. Any terminal observation is subsequent to, and distinct from, quantum recovery.

*Proof.* The complete endpoint is an isometry under the stated construction. Theorem gives recovery. A pure residual makes the syndrome space one dimensional.

None of these gravitational hypotheses is established by the typed fragment or the regulated scalar Hamiltonian. A recovery-during-evaporation claim requires a nonempty intersection

$$I_{\text{geometry}} \cap I_{\text{access}} \cap I_{\text{recovery}}$$

in the same model. A horizonless solution does not itself establish complete residual independence, and a CPTP decoder does not construct its coherent radiation receiver.

The scalar evaporation model of Gaikwad, Kaushal, Mandal, and Wadia has its own low-energy, large-*N*, and preparation assumptions [GaikwadEtAl]. Its geometrical conclusions are not attached to the present decoder without a same-model comparison theorem.

## Conclusions and release boundary

The proved results are:

• complete unitary code preservation and the global restore map ;

• exact recovery precisely when the complete complement is constant, with decoder ;

• reference-preserving CPTP semantics and termination for the declared finite acyclic fragment;

• the record-only obstruction $1 - d^{-1}$;

• equality of unobserved residual readout-and-drop with the pre-existing radiation marginal;

• a complete readout dilation retaining the omitted information;

• finite-time complete-channel approximation and separate qubit dictionaries for the regulated microscopic model;

• dimension-explicit quantum-return certificates and subsequent record comparisons;

• the exact four-Majorana formulas –;

• access-preserving endpoint continuity and fixed-proxy recurrence.

toprule Plan Implemented result and boundary

midrule A Coherent preservation, local quantum decoding, and optional terminal observation have distinct typed interfaces. No live residual wire is silently deleted.

B The global restore map and complete apparatus ledger are explicit. Residual readout-and-drop is external to the autonomous Hamiltonian.

C The destination table includes the exact control and the energy and entropy counterexamples. A nonconstant residual is not equated with residual-only recovery.

D Endpoint identifications preserve access. Fixed finite proxies recur. Syntactic termination is not physical evaporation.

E No field, metric, admitted analytic quantum coefficient, finite-gate compilation theorem, or OpenAI catalogue result is imported from typing or readout.

bottomrule

Complete unitary dynamics preserves the diary in the complete output. Which marginal recovers it is a separate channel question. Destructive readout is a classical-output instrument; discard is further operational forgetting. Neither operation is a theorem of physical annihilation.

The source supplies no positive noncommuting interacting recovery certificate, physical field-to-memory receiver, quantum-gravity unitarity theorem, or complete gravitational evaporation endpoint. The gravitational conclusion is exactly Corollary , with its hypotheses left explicit.

## Declarations

**Verification.** The mathematical claims are the stated analytical arguments. Successful compilation, executed numerical optimization, proof-assistant verification, independent technical review, and journal acceptance are not asserted.

**Source status.** Published external results are identified by ordinary bibliographic records. Cheung companion manuscripts and OpenAI family descriptions are identified as supplied unpublished records or excerpts. The latter are not represented as independently verified published articles. Responsible authorship and archival accessibility of the unpublished records should be confirmed before submission.

**Scope.** No novelty claim is made for exact correction, no-hiding, Stinespring dilation, entanglement-breaking channels, or the information–disturbance theorem. Documentary continuation, comparison, and separation relations do not automatically make one manuscript a premise of another.

appendix

## Additional channel debugging checks

### Populations and averaged realizations

Computational dephasing agrees with the identity on all computational populations but annihilates off-diagonal matrix units. It is not a quantum-code-preserving channel.

For a qubit, the two channels

$$\mathscr{C}_0(\rho) = \mathrm{diag}\,(\rho), \qquad \mathscr{C}_1(\rho) = X\mathrm{diag}\,(\rho)X$$

have constant average *Tr (ρ)I_2/2*, although each frozen realization perfectly distinguishes the computational basis inputs. Constancy of an averaged channel is not constancy of every realization.

If a realization-dependent certificate satisfies $\mathbb{E}_Je_J(t) \le a(t)$, Markov's inequality gives, with probability at least $1 - p$,

$$\delta_{\mathrm{full}}_\mathbf{J}(t) \le \min\{1, d\,a(t)/p + b_{\mathbf{J}}\}.$$

Here *b_J* is a valid complete-channel comparison allowance for that realization. The decoder's dependence on the realization must be declared. A uniform time interval requires a separate uniform estimate.

### A coherent cutoff is not discarded probability

In a common three-dimensional output space, put

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \qquad \sigma = \frac{1}{2}\mathrm{kb}\,00 + \frac{1}{2}|\perp\rangle\langle\perp|.$$

The difference has eigenvalues

$$\frac{1+\sqrt{5}}{4}, \qquad \frac{1-\sqrt{5}}{4}, \qquad -\frac{1}{2}.$$

Therefore

$$d_{\mathrm{tr}}(\mathrm{kb}+{+},\sigma) = \frac{\sqrt{5}+1}{4} > \frac{1}{2}.$$

A discarded weight is not a generic coherent-channel allowance. The flagged-cutoff comparisons in [CheungEn02E] retain their commuting-state assumptions. The regulator used here instead compares complete prepared isometries.

**The covariance convention**
With conjugate-linear annihilation,

$$e^{\mathrm{i}tH_B}\Phi(f)e^{-\mathrm{i}tH_B} = \Phi(e^{\mathrm{i}th_B}f),$$

and

$$\langle\Omega, \Phi(e^{\mathrm{i}sh_B}f_a)\Phi(e^{\mathrm{i}uh_B}f_b)\Omega\rangle = \delta_{ab}\int_{\mu}^{\Lambda}\frac{e^{-\mathrm{i}\omega(s-u)}}{2\omega}\mathrm{d}\omega.$$

The Weyl amplitudes in Section use this convention. A prescribed scalar history is a comparison input, not a replacement for the complete matrix-unit channel of a noncommuting source.

**No model splice**
The quadratic fermionic endpoint and Bessel-type survival law in [CheungHawk02D] belong to that Hamiltonian. No reduction of the quartic scalar Hamiltonian to that model is proved here.

Likewise, a classical mixing estimate, entropy inequality, area law, capacity result, or scalar energy estimate becomes a complete-residual recovery certificate only after a theorem at the named reference-preserving channel interface.

## Documentary analytic separations

This appendix records the scope of the supplied excerpts; it does not attempt the distinct distributional construction appearing in them. Their variables, carrier, signs, cutoffs, and observable are not identified with *C*, *R*, *B*, or with .

**Internal reconstruction is not characteristic composition**
The supplied XIV-H release records an unresolved internal transition-amplitude requirement at H2. XIV-I subsequently constructs its specified internal retarded-pole amplitude module at I2, but leaves the complete joint characteristic operator map unresolved at I3 [CheungXIVH] [CheungXIVI].

The supplied XIV-J excerpts add reconstruction descent and a supported-sector reference for *K_6 + K_8 + K_9* away from the characteristic locus. They do not construct the full map for the six-branch sum on tests meeting that locus [CheungXIVJ]. Neither a normal-jet formula away from a locus nor a conditional coefficient-extension interface supplies an extension across it.

The elementary descent condition can be stated without importing any of those analytic constructions.

**Lemma.** Let *Rec: P → K* be a surjective linear reconstruction map and let *M̃: P → D* be linear. There is a unique linear *M: K → D* with *M̃ = MRec* if and only if

$$\ker \mathrm{Rec} \subseteq \ker \widetilde{M}.$$

*Proof.* Necessity follows by applying the factorization to a null presentation. Conversely, define *M(Rec p) = M̃(p)* . Two presentations of the same reconstructed object differ by an element of *ker Rec*, so the definition is well defined. Surjectivity gives uniqueness.

This is the locally proved algebraic interface in [CheungXIVJ]. It constructs no distributional product.

For example, *qδ^{'}(q) = −δ(q)*. Thus two separately nonzero presentation terms can reconstruct zero. The supplied XIV-J null-presentation check consequently qualifies presentation-degree bookkeeping without

invalidating XIV-I's internal reconstruction or frequency estimates [CheungXIVJ]. An intrinsic joint-symbol claim would require the relevant descent and composition theorems.

### The uncompleted admission chain

The supplied excerpts place the remaining obligations in the order

$$\begin{gathered}\text{full joint characteristic map}\\ \to \text{ joint section and controlled remainder}\\ \to \text{ uniform transverse estimates}\\ \to \text{ contact compatibility}\\ \to \text{ complete output wavefront theorem}\\ \to \text{ admission of the stated quantum coefficient.}\end{gathered}$$

These arrows are obligations, not automatic consequences. An internal frequency estimate is not the transverse estimate of the later quantum composition.

Under the quoted admission contract, the unasserted line would include

$$Q_H R_1^{(1)} \in \mathcal{E}_H.$$

These symbols are documentary identifiers for that separate problem, not new objects constructed in this paper. No such coefficient is admitted here, and no Ward defect is formed from an absent coefficient.

The phrase ``XV-B remains closed'' refers to its admission-dependent exit. It does not deny that XV-B contains abstract interface theorems. Its supplied release explicitly does not construct the inherited full characteristic contraction [CheungXVB]. Neither the finite typed fragment nor the channel certificates above open that exit.

The supplied XVI-N obstruction is conditional on its particular massless instance and stated route assumptions [CheungXVIN]. It is not the missing H2, I3, or J2 construction, does not discharge a universal analytic completion, and does not open the XV-B exit.

The quoted analytic continuation is distinct from the relative classical XIV-A–XIV-B branch. XIII-E's alternative route is not imported. Later continuation labels or documentary patch sheets are not substituted for the required analytic maps. These statements concern the supplied source boundaries; they are not universal nonexistence theorems.

### The carrier-to-record boundary

Landmark I supplies finite destructive measurement semantics within its declared class [CheungI]. It does not define a missing characteristic product. Even after a coefficient is admitted in its stated output space, a later measurement claim would still need a representation, a finite encoding, a specified instrument, an input class, an approximation topology, and an error allowance. Equality of outcome effects alone is weaker than equality of the complete selective instrument.

In the supplied catalogue terminology, O.1 remains NOT_CLOSED. This is a documentary status label: no independent O.1 construction is asserted or discharged by this paper.

## OpenAI catalogue entries: separations only

The supplied companion record attributes the following family descriptions to OpenAI. That attribution and their mathematical claims are not independently verified here [SuppliedCatalogue]. No family is a premise, an evaluation of the present typed program, or a certificate for the four-Majorana control.

In particular, *Family 279 by OpenAI*, described as ``Exact quantum factoring over a fixed finite gate set,'' is not an exact-compilation theorem for our admitted unitary constants. *Family 227 by OpenAI*, described by Sherrington–Kirkpatrick mixing, phase-transition, and critical-autocorrelation topics, is not a scrambling estimate or out-of-time-order evaluation of . The control proof uses neither family. O.1 stays NOT_CLOSED.

toprule Family Supplied documentary topic What is not imported

midrule 279 Exact quantum factoring over a fixed finite gate set Compilation, uniformity, complexity, or implementation of *U_{body, t}*.

227 Sherrington–Kirkpatrick dynamical transition, mixing, and critical autocorrelations Quantum scrambling, an OTOC bound, or a diary-recovery estimate for the source–scalar model.

274 Moore's parity conjecture for *QAC^0* A limitation on this differently specified finite fragment, or an evaluation of its black-hole interpretation.

276 Classical capacity of qubit generalized amplitude-damping channels A complete-complement decoder or a substitute for Theorem .

119 Courtade–Kumar and Hellinger-contraction topics on the discrete cube A quantum destination map, residual-independence certificate, or gravitational conclusion.

bottomrule

The table identifies supplied topics without endorsing their asserted theorems. Similar terminology does not identify models, gate sets, channels, access resources, or observables. No catalogue entry admits the analytic coefficient in Appendix , opens XV-B's admission exit, or discharges the separate XVI-N obligations.

## Archival requirements for a future positive recovery claim

A positive interacting quantum-recovery result should archive:

• the frozen source array, normalization, code, preparation, carrier ordering, couplings, cutoffs, and packet functions;

• regulator choices and their validated complete-channel allowance;

• the exact finite Hamiltonian and controlled propagator evaluation;

• all accessible and complementary logical matrix-unit outputs;

• a rigorously feasible replacement state or complete CPTP decoder;

• positivity, trace-preservation, trace-norm, and arithmetic checks;

• separate qubit dictionaries and invalid-subspace completions;

• a quantum-return program and any distinct terminal-readout program;

• the complete measurement, failure, work, and discard ledger;

• a certified finite time or uniform interval;

• any physical receiver comparison at its named channel interface;

• for a gravitational claim, same-model preparation, access, backreaction, geometry, common interval, and endpoint construction. A terminal classical record or finite-program termination proof does not replace these deliverables.

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N. Cheung, *Hawk04-A: Unitary Information Preservation and Radiation Recovery: A Complete-Complement Formulation of the Black-Hole Information Problem*, version 1. Supplied unpublished manuscript Hawk04-A-v1.tex, n.d. Predecessor source record: global restore, exact correction, residual-only no-hiding, four-Majorana control, endpoint continuity, and recurrence. The relevant channel statements are proved locally.

N. Cheung, *Landmark I*, version 2. Supplied unpublished manuscript excerpts, n.d.; source identifier I_v2.tex in the revision record. In particular, thm:main, ``Characterization." Cited for its declared strategy-class Born law and positive-branch destructive residual update, not as a field-preparation or analytic-admission theorem.

N. Cheung, *En03-A*, version 1. Supplied unpublished manuscript excerpts, n.d. Minimal linear circuit interface and documentary source ledger. The finite fragment in this paper is defined independently.

N. Cheung, *Hawk02-D: Autonomous Fermionic Information Release: Complete-Residual Bounds, Scattering Endpoints, and the Limits of an Evaporation Interpretation*, version 1. Supplied unpublished manuscript Hawk02-D-v1.tex, n.d. Quadratic fermionic comparison and its own scattering endpoint; not a premise for quartic scalar asymptotics.

N. Cheung, *Hawk02-E: Coherent Diaries and Complete-Residual Certification in a Regulated Interacting SYK–Scalar Model*, version 1. Supplied unpublished manuscript Hawk02-E-v1.tex, n.d. Architecture, regulator interface, energy-versus-information separation, and fixed-proxy recurrence. The present Hamiltonian and regulator arguments are supplied locally.

N. Cheung, *En02-B*, version 1. Supplied unpublished manuscript excerpts En02-B-v1.tex, n.d. Quantum-return decoder programs, syndrome consumption, declared-module resource bounds, and compatible channel allowances. No resource optimum is transferred to the present decoder.

N. Cheung, *En02-C*, version 1. Supplied unpublished manuscript excerpts En02-C-v1.tex, n.d. Reference-preserving sequential accounting and the distinction between information availability, decoder existence, and execution.

N. Cheung, *En02-E*, version 2. Supplied unpublished manuscript excerpts En02-E-v2.tex, n.d. Finite Gaussian packet states and flagged-cutoff comparisons. Commuting-state cutoff hypotheses are retained.

N. Cheung, *Em03-C*, version 1. Supplied unpublished manuscript excerpts Em03-C-v1.tex, n.d. Joint apparatus interfaces, coherent comparisons, measurement records, and receiver-access boundaries. No local gravitational detector or microscopic diary channel is imported.

N. Cheung, *Landmark XIV-H: Internal kernel relations on the operator route*, version 1. Supplied unpublished manuscript excerpts XIV-H.tex, n.d. In particular, thm:H2 and thm:release. Documentary analytic separation only.

N. Cheung, *Landmark XIV-I*, version 1. Supplied unpublished manuscript excerpts XIV-I.tex, n.d. In particular, thm:I2, thm:I3, and thm:release. Constructed internal amplitude module and unresolved full joint characteristic map; documentary separation only.

N. Cheung, *Landmark XIV-J*, version 1. Supplied unpublished manuscript excerpts XIV-J.tex, n.d. In particular, prop:descent, lem:null, thm:reference, thm:conditionalextension, thm:J2, and cor:noadmission. Supported-sector reference is not a full characteristic-map or admission result.

N. Cheung, *Landmark XV-B*, version 1. Supplied unpublished manuscript excerpts XV-B.tex, n.d. Abstract descent, relative-lifting, contact, and extension interfaces; thm:release does not admit the inherited quantum coefficient.

N. Cheung, *Landmark XVI-N*, version r12. Supplied unpublished manuscript excerpts XVI-N.tex, n.d. In particular, thm:massless-obstruction. Conditional specified-instance obstruction, not a universal analytic or gravitational completion.

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